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ECONOMIC EVALUATION OF INDUSTRIAL PROJECTS

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ABSTRACT

This paper deals with the economic evaluation of industrial projects. Some traditional techniques, which have been used by practicing managers or advocated by academics, become questionable in the face of modern developments in finance theory. The objective of this paper is to examine how these modern developments in finance theory are applied to project evaluation.

The paper is divided into two parts. Part 1 formulates a fundamental procedure for managers to follow in selecting projects and briefly reviews some of the most important developments in modern finance theory. Part 2 examines how these developments are applied to single-period as well as multiperiod project evaluation and reexamines some traditional approaches of project evaluation.

Acknowledgement

My sincere gratitude is to Professor Gabriel R. Bitran. His valuable criticism, help, and encouragement have made this paper all possible.

Many thanks also go to Professor Stewart Myers for his generosity to spend a lot of time with me discussing this paper.

PART 1

INTRODUCTION TO ECONOMIC EVALUATION OF INDUSTRIAL PROJECTS

SECTION 1

DEFINITION, ASSUMPTION , AND FUNDAMENTAL PROCEDURE

Two different approaches have been suggested to evaluate industrial projects: the economic approach and the social approach. A profit-seeking firm usually takes an economic approach to find out what kind of economic impact a candidate project would have on the firm if it were adopted. A government or a non-profit organization, on the other hand, usually takes a social approach to find out how a candidate project would affect the : society in terms of national income, standard of living, unemployment, and so forth. These two approaches are not mutually exclusive. In many cases, an economic evaluation is only part of a social evaluation. An example can be found in [19]. Some large companies have also begun to consider their social responsibility and want to find out the social impact of their projects. However, it is still debatable whether a private profit-seeking firm should modify its original goal, which is to maximize the welfare of its current shareholders, in

order to be socially responsible. Relevant discussions can be found in [15, 33, 34]. In this paper, we regard ourselves as the managers of a private, profit-seeking firm whose shares are publicly traded in capital markets (e.g., the New York Stock Exchange) and concern ourselves only with the economic evaluation of industrial projects.

In a broad sense, a complete procedure for evaluating projects includes: the generation of candidate projects, the estimation of cash flows for candidates, the evaluation of cash flows, and the reevaluation of projects after their acceptance. In this paper, we primarily consider the evaluation of cash flows. The following definition is therefore used in this paper.

(Definition) The economic evaluation of industrial projects is a procedure taken by the managers of a firm to evaluate streams of cash flows associated with the firm's candidate projects so that the managers can accept or reject some of these projects with a view to promote the welfare of the firm's current shareholders.

To evaluate a stream of cash flows is definitely not an easy matter. The initial cash flow may be known with

certainty. But, future cash flows are uncertain in almost all cases. In this paper, we assume that every future cash flow is a random variable. Then, a stream of future cash flows becomes a vector of random variables, which is called a "prospect." There will be a multivariate probability distribution for each prospect.

In the above, we have used the terms "period" and "future" several times. A period can be a week, a month, a year, or any other length of time. The future can extend to an ending point or be endless. To decide how long the future should extend and how long a period should be is either arbitrary or dependent on the particular problem. We can have a one-period problem, a multiperiod problem with a finite horizon, or a multiperiod problem with an infinite horizon. Two periods in the same problem need not have the same length. Let time 0 denote the time decisions are being made. Period 1 extends from time 0 to time 1; period 2 extends from time 1 to time 2, and so forth. We assume events (e.g., cash flows) for any period occur at the end of that period; nothing happens during the period.

For the purpose of this paper, we give a project the following characteristics:

- (1) An initial cash flow $-I_0$ occurs at time 0. This is the only cash flow occurring at time 0. In

most cases, I_0 is a positive number and stands for an initial cash outlay. We assume that I_0 is known with certainty.

(2) Cash flow for period t ($t > 0$), denoted by

$\Delta\tilde{Y}_t$, occurs at time t . $\Delta\tilde{Y}_t$ is a random variable.

Then, a project can be represented by $[-I_0, \Delta\tilde{Y}]$ where I_0 is its initial cash outlay and $\Delta\tilde{Y} = [\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$ is its prospect. There is a multivariate probability distribution associated with $\Delta\tilde{Y}$.

In a sense, a project is an exchange of I_0 for $\Delta\tilde{Y}$. Managers give up an initial cash outlay I_0 (known with certainty) in exchange for a prospect $\Delta\tilde{Y}$ (random) in hopes of promoting the welfare of holders of the firm's current shares. By current shares we mean those shares which were outstanding before time 0 and are still outstanding at time 0.

Let us explain the term "cash flow" in greater details. Equation (1.1) holds for the cash flow in any period t of any project.

$$\begin{aligned}
 (\text{cash flow})_t &= (\text{cash earnings before interest and} \\
 &\quad \text{tax expenses})_t \\
 &\quad - (\text{tax expenses})_t \\
 &\quad - (\text{investment})_t \\
 &\quad + (\text{disinvestment})_t
 \end{aligned} \tag{1.1}$$

Notice that interest expenses and depreciation expenses are tax deductible.

Conceptually, we relate every item in the right hand side of Equation (1.1) with the cash flow to all security holders in period t . Let Y_t denote the cash flow to all security holders in period t .

$$\begin{aligned}
 Y_t = & \text{interest/principal payments to bonds which were out-} \\
 & \text{standing before time } t \text{ and are still outstanding} \\
 & \text{at time } t \\
 & + \text{dividend payments to shares which were outstanding} \\
 & \text{before time } t \text{ and are still outstanding at time } t \\
 & - \text{cash receipts from the sale of new bonds at time } t \\
 & - \text{cash receipts from the sale of new shares at time } t
 \end{aligned}
 \tag{1.2}$$

For any project, cash produced by its (cash earnings before interest and tax expenses) $_t$ and (disinvestment) $_t$ are made available either for interest/principal payments and dividend payments in period t or for reducing the quantities of new bonds/shares issued in period t ; cash consumed by its (tax expenses) $_t$ and (investment) $_t$ are either to reduce interest/principal payments and dividend payments in period t or to increase the quantities of new bonds/shares issued in period t . Suppose a firm has n projects. Then,

$$\sum_{i=1}^n (\text{cash flow of project } i)_t = Y_t \tag{1.3}$$

Equation (1.3) says that the cash flow generated by the whole firm in period t is equal to the cash flow to all security holders in period t . Notice that cash holding is also a form

of production asset, and hence, an increase (decrease) in cash holding is also an investment (disinvestment).

We shall make the following eight assumptions:

(Assumption 1) Any individual's welfare can be represented by the utility he derives from his lifetime consumption stream.

(Assumption 2) The functional relationship between utility and lifetime consumption stream of an individual is known with certainty (i.e., known and independent of the state of nature) to him. He prefers more consumption to less in each period.

(Assumption 3) Any individual is an expected-utility maximizer in determining his lifetime consumption stream.

Resources held by any individual can be classified into three categories: labor, cash, and securities. (We assume that production assets can be held only by firms.) With the

help of labor markets, an individual can exchange his labor for cash at the ruling prices if he wishes to do so. With the help of capital markets, an individual can exchange his holding of securities for cash or his holding of cash for securities at the ruling prices if he wishes to do so. With the help of consumption goods markets, an individual can exchange his holding of cash for consumption goods at the ruling prices if he wishes to do so. Figure 1-1 illustrates the interrelationships among labor markets, capital markets, consumption goods markets, and investors. The resources held by an investor at time 0, together with the employment opportunities (in terms of the probability distribution of labor income) available to him in the labor markets in each future period, the portfolio opportunities (in terms of one-period rate-of-return probability distributions of combinations of securities) available to him in the capital markets in each future period, and the consumption opportunities (in terms of goods and services and their prices) available to him in the consumption goods markets in each future period, will decide the maximized expected utility at time 0 of lifetime consumption stream.

(Assumption 4) Any operating (investment or financing) decision of a firm does not affect the employment opportunities available to an investor in the labor markets in each

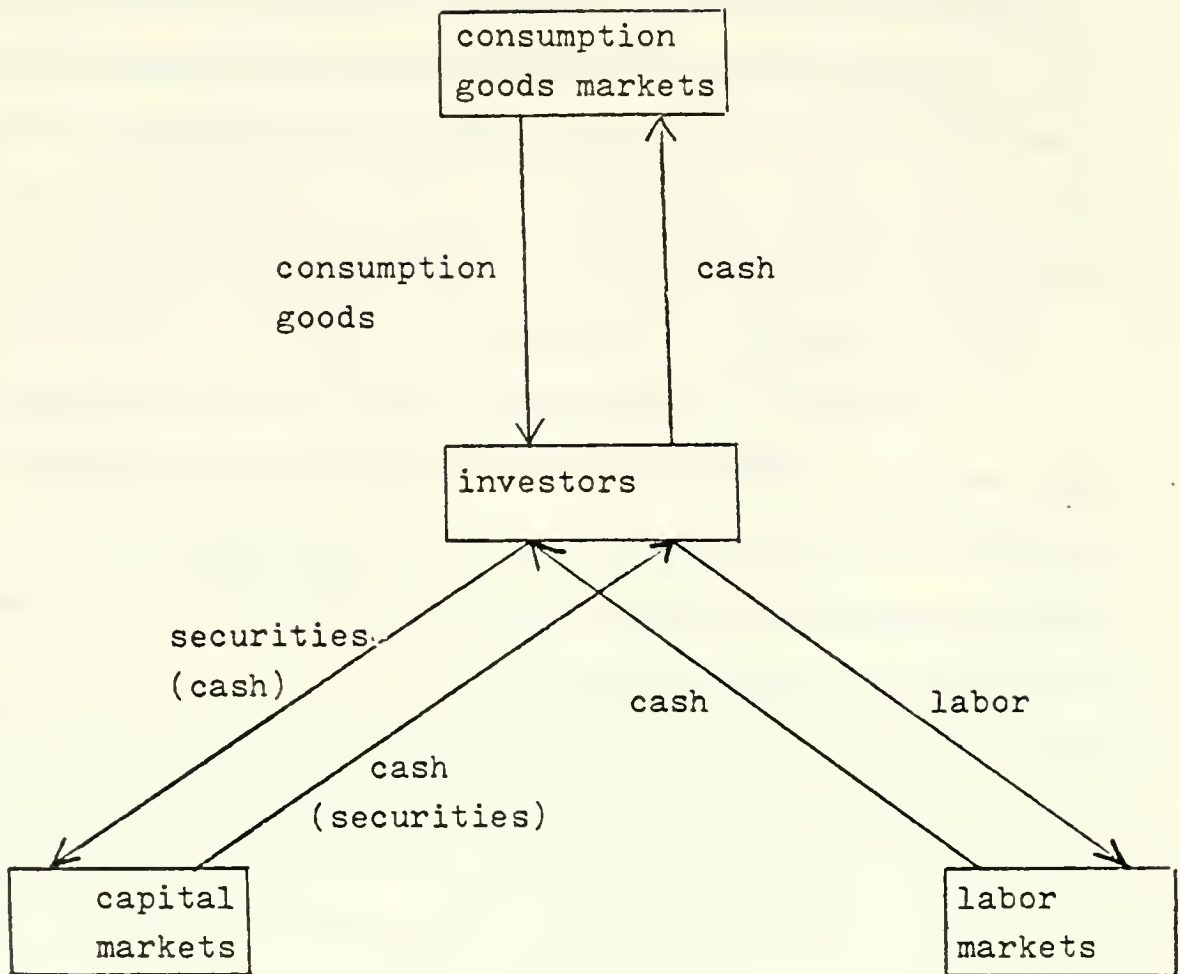


Figure 1-1

future period.

(Assumption 5) Any operating decision of a firm does not affect the consumption opportunities available to an investor in the consumption goods markets in each future period.

(Assumption 6) Capital markets are perfect.

A capital market is perfect if

- (1) there are no brokerage fees, transfer taxes, or other transaction costs incurred when securities are bought, sold or issued;
- (2) all traders (buyers and sellers, or issuers) have equal and costless access to information about relevant properties of the securities;
- (3) all securities are infinitely divisible;
- (4) any operating decision of a firm does not affect the portfolio opportunities available to an investor in the capital market in each future period; in other words, the capital market is complete with respect to operating options open to a firm in the sense that any operating decision of a firm does not create a unique new security which would allow an investor to achieve a previously unobtainable

- portfolio in each future period;
- (5) all traders' activities (buying and selling, or issuing) in the capital market have no detectable effect on the ruling prices; in other words, the traders are price-takers in the capital market;
- and (6) there are always perfect substitutes for any securities of a firm and the firm's operating decisions have no effect on the prices of the securities issued by other issuers.

Under Assumptions 4, 5, and 6-(4), at time 0 managers of a firm can promote a current shareholder's welfare only by increasing the market value of the firm's current shares. It is the so-called Market Value Maximization rule. However, under Assumption 6-(5,6) managers can increase the market value of the firm's current shares only by making wise decisions in operating the firm.

If the capital market is not in equilibrium (that is to say, the market is not cleared by ruling prices), there are pressures for future changes, and it is very difficult to establish relationships between security prices and managers' operating decisions.

(Assumption 7) Capital markets are in equilibrium.

Strictly speaking, the Market Value Maximization rule is still not appropriate when capital markets are not efficient.

A capital market is efficient if (1) equilibrium security prices fully reflect all available information and (2) these prices react instantaneously in an unbiased fashion to new information. For example, GM decides to shut down one of its manufacturing plants. This piece of information will be fully reflected in the price of GM shares or bonds which are traded in an efficient capital market.

(Assumption 8) Capital markets are efficient.

Under Assumptions 1 through 8, managers, who are supposed to make decisions to promote the welfare of the current shareholders are now to make decisions to increase the market value of current shares. (See [1, 2, 7, 8, 9, 12, 23, 30, 38]) Two additional comments are made here. First, by "to increase the market value of current shares" we really mean "to increase the sum at time 0 dividend payment, if any, to current shareholders and time 0 ex-dividend market value of current shares." Second, while we assume capital markets are perfect, efficient, and in equilibrium, we do not assume that product markets are necessarily perfect, efficient, and in equilibrium. If product markets were perfect, efficient and in equilibrium, there would not exist any profitable project.

Now let us introduce the "Value Additivity Principle". Define V_i as the market value of security i which has a prospect $Z = [\tilde{Z}_{i1}, \tilde{Z}_{i2}, \dots, \tilde{Z}_{i\infty}]$, where $\tilde{Z}_{it}$ is the cash payment (a random variable) to holders of security i at time t . The Value Additivity Principle (VAP) says that in a perfect market

$$\text{if } \tilde{Z}_H = \sum_{i=1}^n \tilde{Z}_i, \text{ then } V_H = \sum_{i=1}^n V_i \quad \text{at equilibrium}$$

where V_H is the market value of $\tilde{Z}_H$. The VAP holds under the assumption of a perfect capital market because of the mechanism of "arbitrage", which can be performed by either the professional arbitragers or the investors. A more detailed description of the VAP can be found in [17].

Let $\Delta\tilde{Y}$ denote $[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$, the prospect of a project. Suppose that there exists a security in the market such that this security has $\Delta\tilde{Y}$ as its prospect, and has a market value of $V_{\Delta Y}$. Let I_0 denote the initial cash outlay of the project. It can be shown that under the assumption of the old bond value being unchanged, the value of the current (i.e., old) shares will be changed by $\Delta V - I_0$ if the project is adopted where ΔV is the change in firm value due to the adoption of the project. But under the VAP, we have $\Delta V = V_{\Delta Y}$. Then, managers should adopt the project if

$$V_{\Delta Y} - I_0 > 0 \tag{1.4}$$

The assumption of the old bond value being unchanged in spite of the adoption of the project is usually made in finance theory in order to simplify the discussion.

Now we are in a good position to outline a fundamental procedure for managers to follow in selecting projects. The procedure includes the following steps:

- (1) Find out all possible alternatives within a project. One and only one alternative will be chosen to represent the project.
- (2) Choose the alternative with the highest value of $(V_{\Delta Y} - I_0)$ to represent the project.
- (3) Carry out the project if its representative alternative has a positive value of $(V_{\Delta Y} - I_0)$.

Sometimes when we refer to a project, we actually refer to its representative alternative.

The final criterion obtained, $V_{\Delta Y} - I_0$, indicates the difficulty with the economic evaluation of industrial projects. How can we determine $V_{\Delta Y}$? That is the topic in the remainder of the paper.

So far we have made many assumptions and we shall make more assumptions later. We certainly do not expect all these assumptions to be exactly true in the real world. However, abstraction is required in building any theory or model. Any real-world problem of interest is usually very complex; an analysis of all the relevant aspects is extremely difficult,

if not impossible. We just hope that those theories or models we build can perform reasonably well in a real world.

SECTION 2

PORTFOLIO THEORY: A BRIEF REVIEW

Modern developments in financial and investment management have made questionable some techniques which have long been used in evaluating projects. Many of these modern developments have the portfolio theory as their cornerstone. Portfolio theory is also called the Markowitz theory because it was first introduced by Markowitz [26, 27].

Portfolio theory tells investors how to make decisions in choosing securities whose future prospects are not known with certainty. Securities, as used in the portfolio theory, include not only those financial securities issued by firms, but also other capital assets (e.g., a house), jobs, husbands, wives, etc. Any such security involves a prospect which is not known with certainty. Any combination of such securities is called a portfolio. By definition, a portfolio can be composed of only one security.

Portfolio theory deals with a one-period problem, no

matter how long a period is. The performance of a portfolio is measured by its rate of return. In particular, for securities of firms, the rate of return can be defined as $[(\text{end-of-period price}) - (\text{beginning-of-period price}) + (\text{cash payment to security holder})] / (\text{beginning-of-period price})$; other definitions are also available (see [14]).

Portfolio theory makes the following assumptions:

- (1) An investor is willing to reduce a multiperiod consumption investment decision to a one-period decision involving the first period's consumption and terminal wealth at the end of the first period. That is to say, an investor is willing to act as if he is solving a one-period problem.
- (2) An investor is willing to view the rate of return of any security as a random variable. With regard to the (subjective) probability distribution of the random variable, an investor concerns himself only with its expected value (E), standard deviation (σ), and covariance with the rate of return of any other security.
- (3) Once all possible portfolios have been found, an investor is willing to choose among portfolios solely

on the basis of two parameters -- expected value and standard deviation of the rate of return. That is to say, two portfolios with quite different rate-of-return probability distributions but with the same expected value and standard deviation are equivalent for the purpose of an investor.

- (4) An investor prefers less σ to more (i.e., he is risk-averse), and prefers more E to less. This implies that an investor tries to hold his portfolio in such a way that σ is minimized for a given E and E is maximized for a given σ .

These assumptions may not be true in a real world. But, as mentioned before, such abstraction is required in building a theory. We hope that the portfolio theory is able to help an investor making the right decision.

The reason why an investor combines several securities together to form a portfolio comes from the assumption that he is risk-averse. One extreme example is given as follows: Let the rate-of-return probability distributions of two securities, i and j , have the same expected value E and the same standard deviation σ ; also let ρ denote the correlation coefficient between them. Suppose we put one half of our fund into each of these two securities to form a portfolio. Then, the expected return of the new portfolio is still E ; but the standard deviation of the new portfolio becomes

$(\frac{1}{2} \sigma^2 + \frac{1}{2} \rho \sigma^2)^{\frac{1}{2}}$. As long as $\rho < 1$,

$(\frac{1}{2} \sigma^2 + \frac{1}{2} \rho \sigma^2)^{\frac{1}{2}} < \sigma$. In particular, when $\rho = -1$,

$(\frac{1}{2} \sigma^2 + \frac{1}{2} \rho \sigma^2)^{\frac{1}{2}} = 0$. The phenomenon that less perfectly correlated securities are combined together in order to reduce risk is called "diversification."

Given all the possible portfolios, the portfolio theory tells an investor to concern himself only with some specific portfolios. These specific portfolios are called efficient portfolios. An efficient portfolio is a portfolio which has a smaller σ than any other portfolio with the same E and has a larger E than any other portfolio with the same σ . Let a portfolio p be represented by the expected value (E_p) and standard deviation (σ_p) of its rate of return. And let r_f be the rate of return on a riskless security, where r_f is known with certainty.

In Figure 2-1 and Figure 2-2, the efficient portfolios are indicated by a bold line. Figure 2-1 shows a case where no riskless security exists; the dotted area is the set of all possible portfolios formed by risky

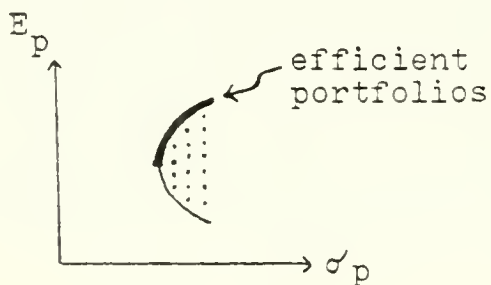


Figure 2-1

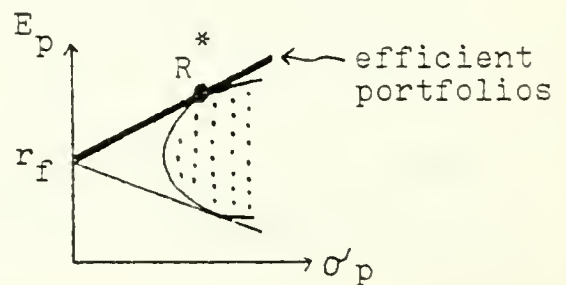


Figure 2-2

securities. Fig. 2-2 shows a case where a riskless security exists; the dotted area is the set of all possible portfolios formed by risky securities; and the triangle-shaped area is the set of all possible portfolios formed by risky securities and the riskless security. A detailed explanation on how to determine the E_p and σ_p of any possible portfolio, in particular how to determine the E_p and σ_p of all efficient portfolios, and why the curve representing the set of efficient portfolios is always convex (possibly linear) to the upper-left corner can be found in [26] and [27].

Once the efficient portfolios have been determined, an investor should choose one among them according to his preference.

In Figure 2-2, portfolio R^* represents the efficient portfolio involving only risky securities; all other efficient portfolios, except point i , have the following characteristic: the ratio of the amount invested in risky security A to that in risky security B is equal to the same ratio in portfolio R^* . Portfolio R^* is called the optimal combination of risky securities.

As shown in Figure 2-2, the existence of a riskless security will cause the optimal combination of risky securities to exist, and then greatly simplify the task of portfolio selection. If an investor decides to invest some of his fund in risky securities,

there is only one appropriate combination of risky securities available for him regardless of his preference. The consideration of an investor's preference and the consideration of an optimal combination of risky securities is separated. It is called the separation theorem. Notice that the separation theorem does not hold when there does not exist a riskless security.

SECTION 3

CAPITAL ASSET PRICING MODEL:

A BRIEF REVIEW

Capital Asset Pricing Model (CAPM) describes the relationships between prices and other elements. It makes numerous assumptions which obviously do not hold in a real world. Again, abstraction is required in building the model. We just hope its implication or prediction is a good approximation to the reality. The assumptions made by the CAPM include:

- (1) Each investor is a Markowitz-efficient investor. By "Markowitz-efficient investor" we mean an investor who does what Markowitz theory (i.e., portfolio theory) tells him to do; that is to say, a Markowitz-efficient investor is an investor who holds an efficient portfolio. This assumption implies that all the assumptions made in portfolio theory are also made here.
- (2) Each investor has the same one-period horizon as any other investor.
- (3) Each investor can borrow or lend any amount of money as he wishes at a riskless rate of return. There is only one riskless rate of return no matter who borrows or who lends.
- (4) All investors have identical (subjective) estimates of

the expected values, standard deviations and covariances of rate of return among all That is to say, they hold "homogeneous expectations."

(5) Markets are perfect.

(6) Markets are in equilibrium, which is reached through a process of tâtonnement with recontracting.

Under these assumptions, all investors have the same set of efficient portfolios. That is to say, the bold line in Figure 3-1 which indicates the set of efficient portfolios is the same for all investors. This line is called the

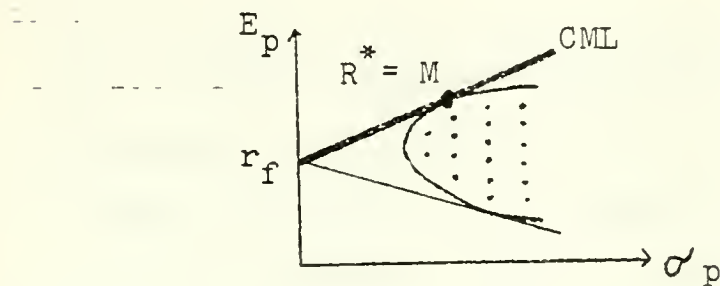


Figure 3-1

Capital Market Line (CML). Since portfolio R^* , the optimal combination of risky securities, is also the same for all investors, it deserves another name. We call it the market portfolio, denoted by M . Because each investor is a Markowitz-efficient investor, he is to select a portfolio along the CML. Any portfolio along the CML can be obtained by some combination of the market portfolio M and the riskless security. And any two portfolios (except the riskless one) along the CML are perfectly correlated.

Now, the question facing an investor turns out to be how to allocate his fund between the market portfolio M and the riskless security. This question can only be answered by an investor's preference. The CAPM, based on the portfolio theory, just says that an investor is to select a portfolio along the CML. The equation for the CML is

$$E_p = r_f + \left(\frac{E_M - r_f}{\sigma_M} \right) \sigma_p \quad (3.1)$$

where E_M and σ_M are the expected value and standard deviation, respectively, of the rate of return on the market portfolio M. In equilibrium, the market portfolio M must include all risky securities.

Equation (3.1) is true only for efficient portfolios. We need another formula for inefficient portfolios. Let i be any (efficient or inefficient) portfolio. It has been shown [36] that the curve standing for the portfolios, which are composed of portfolio i and the market portfolio M only, is tangent to the CML at M; and consequently a representative form for the CAPM can be derived:

$$E_i = r_f + \left(\frac{E_M - r_f}{\sigma_M^2} \right) \text{cov}(\tilde{r}_i, \tilde{r}_M) \quad (3.2)$$

where $\tilde{r}_i$ = rate of return on portfolio i

$\tilde{r}_M$ = rate of return on portfolio M

$\text{cov}(\tilde{r}_i, \tilde{r}_M)$ = covariance of $\tilde{r}_i$ and $\tilde{r}_M$

Equation (3.2) is illustrated in Figure 3-2. We call the

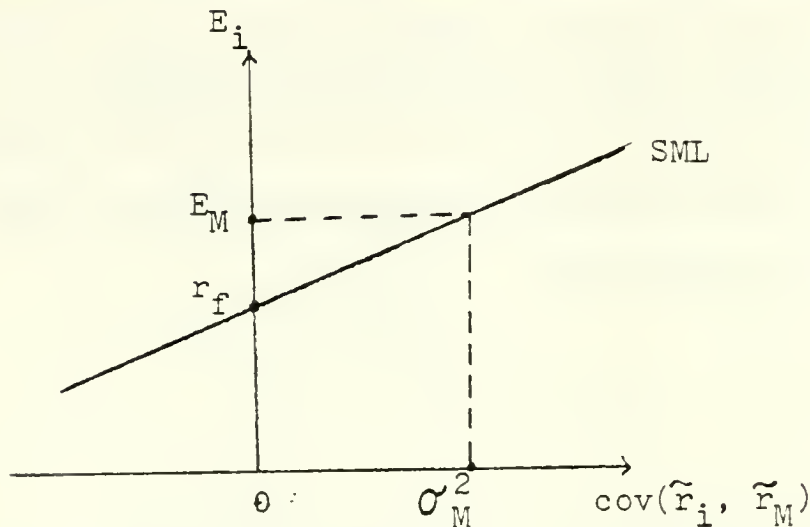


Figure 3 -2

line standing for Equation (3.2) the Security Market Line (SML). Equation (3.2) holds for any efficient portfolio, any inefficient portfolio, or any security. Notice that when $\text{cov}(\tilde{r}_i, \tilde{r}_M) < 0$, $E_i < r_f$.

The CAPM has some convenient forms, in addition to Equation (3.2). Let ρ_{pM} denote the correlation of $\tilde{r}_i$ and $\tilde{r}_M$; then Equation (3.2) becomes

$$E_i = r_f + \left(\frac{E_M - r_f}{\sigma_M} \right) \rho_{iM} \sigma_i \quad (3.3)$$

Or, let

$$\lambda_i = \text{cov}(\tilde{r}_i, \tilde{r}_M) / \sigma_M^2 \quad (= \rho_{iM} \sigma_i / \sigma_M) \quad (3.4)$$

then Equation (3.2) becomes

$$E_i = r_f + (E_M - r_f) \lambda_i \quad (3.5)$$

Equations (3.3) and (3.5) are illustrated in Figures 3-3

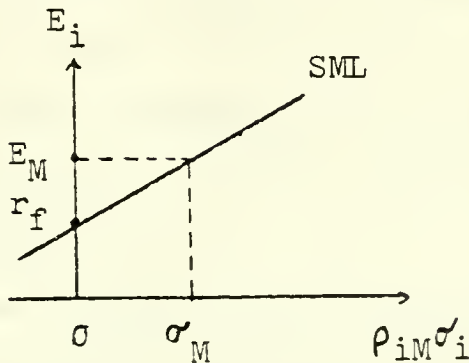


Figure 3-3

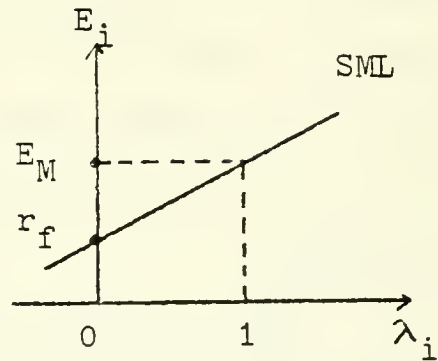


Figure 3-4

and 3-4, respectively. In equilibrium, every portfolio or security will plot along the SML; but only efficient portfolios will plot along the CML.

The CAPM says that the expected return of any portfolio is a time return plus its own risk return; the time return is the riskless rate of return r_f , which is the same for any portfolio, and its own risk return is its covariance with

the market portfolio M multiplied by a constant

$(\frac{E_M - r_f}{\sigma_M^2})$. In other words, the SML says that the risk

premium, the expected return minus the riskless rate of return, of any portfolio is its covariance with M multiplied by a constant $(\frac{E_M - r_f}{\sigma_M^2})$. As a result, the correct risk

measure for any portfolio i becomes the cov $(\tilde{r}_i, \tilde{r}_M)$, or $\rho_{iM}\sigma_i$, or λ_i . The standard deviation σ_i of an inefficient portfolio i is not a legitimate risk measure any more; only a portion of it, $\rho_{iM}\sigma_i$, represents the correct risk and the remaining portion of it, $(1 - \rho_{iM})\sigma_i$, can be diversified away. For an efficient portfolio i, the standard deviation σ_i is still a legitimate risk measure since in this case $\sigma_i = \rho_{iM}\sigma_i$.

The immediate problem when we apply the CAPM in practice is how to find the market portfolio M, which is supposed to include all shares, bonds, houses, machines, jobs, etc. Inevitably, we have to use a proxy like Standard & Poor's 500-stock index, or Fisher index included in CRISP (Center for Research in Security Prices at the University of Chicago), or the rate of return on some well-constructed portfolio which fairly represents the "real" market portfolio. For any such proxy, we still call it the market portfolio M in the CAPM setting; and we assume that the CAPM still holds when a proxy is used. A relevant

discussion on the problems involved in using a proxy can be found in [31].

Estimate of λ_i for a firm's stock i can be obtained from past data using ordinary-least-square regression. The 3-month U.S. Treasury Bill is usually taken as the riskless asset. The regression equation used (see [23]) is

$$(r_{it} - r_{ft}) = \alpha_i + \lambda_i(r_{Mt} - r_{ft}) + e_{it} \quad (3.6)$$

where $E(e_{it}) = 0$ for each period t

$$\text{var}(e_{it}) = \sigma_i^2 \quad \text{for each period } t$$

$$\text{cov}(e_{it_1}, e_{it_2}) = 0 \quad \text{for } t_1 \neq t_2$$

Equation (3.6) implicitly assumes that λ_i is unchanged over time. Theoretically speaking, α_i should be zero. However, using past data to approximate the true relationship between $(r_{it} - r_{ft})$ and $(r_{Mt} - r_{ft})$, α_i might differ from zero if one of the assumptions made by the CAPM is not valid in the real world. Notice also, the least-square estimate

$$\hat{\lambda}_i = \frac{\sum_t (r_{it} - r_{ft} - \overline{r_i - r_f})(r_{Mt} - r_{ft} - \overline{r_M - r_f})}{\sum_t (r_{Mt} - r_{ft} - \overline{r_M - r_f})^2}$$

where $\overline{r_i - r_f}$ = average of all observed $(r_{it} - r_{ft})$

$\overline{r_M - r_f}$ = average of all observed $(r_{Mt} - r_{ft})$

$\overline{r_i}$ = average of all observed r_{it}

and $\overline{r_M}$ = average of all observed r_{Mt}

If r_{ft} = constant for all t , then

$$\hat{\lambda}_i = \frac{\sum_t (r_{it} - \overline{r_i})(r_{Mt} - \overline{r_M})}{\sum_t (r_{Mt} - \overline{r_M})^2}$$

However, the situation in which r_{ft} = constant for all t is not likely to occur in empirical work because r_{ft} varies from period to period when observed empirically. Therefore, we shall observe in almost all cases that

$$\hat{\lambda}_i \neq \frac{\sum_t (r_{it} - \overline{r_i})(r_{Mt} - \overline{r_M})}{\sum_t (r_{Mt} - \overline{r_M})^2}$$

However, the difference is statistically insignificant [25]. Consequently, we can continue using Equation (3.6) as our regression equation in estimating λ_1 .

PART 2
PROJECT EVALUATION MODELS

SECTION 4

CAPM IN SINGLE-PERIOD PROJECT EVALUATION

Let $(-I_0, \tilde{\Delta Y}) = (-I_0, \tilde{\Delta Y}_1, \tilde{\Delta Y}_2, \dots, \tilde{\Delta Y}_\infty)$ denote the stream of cash flows of a project (or its representing alternative). Under several assumptions made in Part 1, we can say that managers should accept this project if $V_{\Delta Y} - I_0 > 0$ where $V_{\Delta Y}$ is the market value $\tilde{\Delta Y}$ would assume, should $\tilde{\Delta Y}$ be publicly traded in a capital market. Obviously, the question facing us is how to find $V_{\Delta Y}$.

Suppose a security i is traded in a capital market. Let $\tilde{Z}_i$ be the only future cash flow it has. This cash flow occurs at the end of period 1 (i.e., time 1) and includes its end-of-period market value and any cash payment during period 1. Let V_i be its current (time 0) market value. By definition, the rate of return

$$\tilde{r}_i = (\tilde{Z}_i - V_i)/V_i = \tilde{Z}_i/V_i - 1 \quad (4.1)$$

and,

$$E(\tilde{r}_i) = E(\tilde{Z}_i)/V_i - 1 \quad (4.2)$$

The CAPM says, under several assumptions,

$$E(\tilde{r}_i) = r_f + \left(\frac{E_M - r_f}{\sigma_M^2} \right) \text{cov}(\tilde{r}_i, \tilde{r}_M) \quad (4.3)$$

where M is the market portfolio or its surrogate.

Let

$$\phi = \frac{E_M - r_f}{\sigma_M^2} \quad (4.4)$$

Then, Equation (4.3) becomes

$$\frac{E(\tilde{Z}_i) - V_i}{V_i} = r_f + \phi \frac{\text{cov}(\tilde{Z}_i, \tilde{r}_M)}{V_i} \quad (4.5)$$

After rearranging, Equation (4.5) becomes

$$V_i = \frac{E(\tilde{Z}_i) - \phi \text{cov}(\tilde{Z}_i, \tilde{r}_M)}{1 + r_f} \quad (4.6)$$

Equation (4.6) is also a form of the CAPM. We can derive another form of the CAPM from Equation (4.6). Let $\tilde{e}_i$ be such that

$$\tilde{Z}_i = E(\tilde{Z}_i)(1 + \tilde{e}_i) \quad (4.7)$$

or

$$\tilde{e}_i = \tilde{Z}_i / E(\tilde{Z}_i) - 1 \quad (4.8)$$

where

$$E(\tilde{e}_i) = 0 \quad (4.9)$$

Then,

$$\text{cov}(\tilde{Z}_i, \tilde{r}_M) = E(\tilde{Z}_i) \text{cov}(\tilde{e}_i, \tilde{r}_M) \quad (4.10)$$

Substituting Equation (4.10) into Equation (4.6), we have

$$V_i = \frac{E(\tilde{Z}_i) [1 - \phi \text{cov}(\tilde{e}_i, \tilde{r}_M)]}{1 + r_f} \quad (4.11)$$

There is an ^{interesting} Δ interpretation for Equation (4.6) or (4.11). The numerator of the right-hand side of either equation can be regarded as the certainty equivalent of $\tilde{Z}_i$ and therefore is divided by a riskless discount factor $(1 + r_f)$ to obtain the present value of $\tilde{Z}_i$. This interpretation is related to the Certainty Equivalent (CE) model to be discussed later.

From Equation (4.2), we have

$$V_i = \frac{E(\tilde{Z}_i)}{1 + E(\tilde{r}_i)} \quad (4.12)$$

$$\text{where } E(\tilde{r}_i) = r_f + \phi \text{cov}(\tilde{r}_i, \tilde{r}_M) \quad (4.13)$$

Substituting Equation (4.6) into Equation (4.12), we have

$$\frac{E(\tilde{Z}_i)}{1 + E(\tilde{r}_i)} = \frac{E(\tilde{Z}_i) - \phi \text{ cov } (\tilde{Z}_i, \tilde{r}_M)}{1 + r_f}$$

$$E(\tilde{r}_i) = \frac{1 + r_f}{1 - \phi \text{ cov } (\tilde{Z}_i/E(\tilde{Z}_i), \tilde{r}_M)} - 1 \quad (4.14)$$

From Equation (4.10), we have

$$\text{cov } (\tilde{Z}_i/E(\tilde{Z}_i), \tilde{r}_M) = \text{cov } (\tilde{e}_i, \tilde{r}_M) \quad (4.15)$$

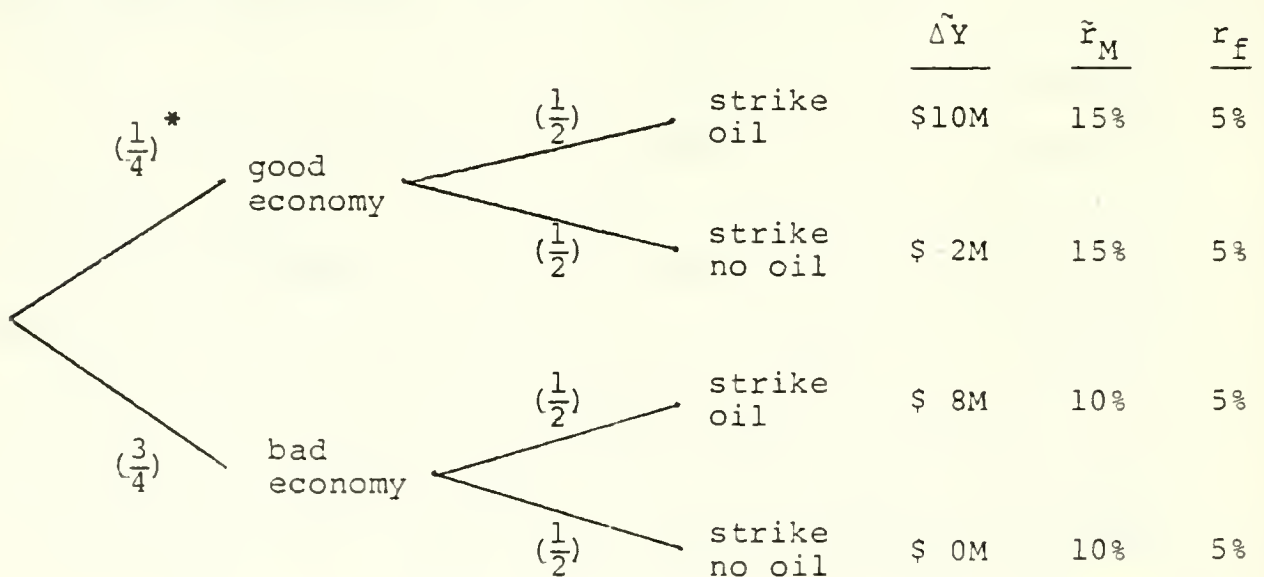
Substituting Equation (4.15) into (4.14), we have

$$E(\tilde{r}_i) = \frac{1 + r_f}{1 - \phi \text{ cov } (\tilde{e}_i, \tilde{r}_M)} - 1 \quad (4.16)$$

To use Equation (4.12), we need know $E(\tilde{r}_i)$ first. Equations (4.13), (4.14) and (4.16) are all equivalent expressions for $E(\tilde{r}_i)$. But $\tilde{r} = \tilde{Z}_i/V_i - 1$ and V_i is exactly what we are looking for; therefore Equation (4.13) is usually not a convenient formula to work with. Through Equation (4.14) or (4.16), we can work on $\tilde{Z}_i$ directly. There is also an interesting interpretation for Equation (4.12). The denominator of this equation, $1 + E(\tilde{r}_i)$ is a risk-adjusted factor and therefore is to divide the expected cash flow of $\tilde{Z}_i$ in order to obtain the present value of $\tilde{Z}_i$. This interpretation is related to the Risk Adjusted Discount (RAD) model, to be discussed later.

Now suppose a single-period project has $[-I_0, \tilde{\Delta Y}]$ as its stream of cash flows. Then, $V_{\Delta Y}$ can be obtained by using Equation (4.6), (4.11), or (4.12) by letting $\tilde{Z}_i = \tilde{\Delta Y}$ and $V_i = V_{\Delta Y}$. Managers should accept this project if $V_{\Delta Y} - I_0 > 0$. In order to use Equation (4.6), (4.11), or (4.12), managers (or some other experts working for managers) must give their (somewhat subjective) judgement on some characteristics of the probability distributions of $\tilde{\Delta Y}$ and $\tilde{r}_M$. Presented below is a scenario approach.

Suppose a single-period project needs \$5M (known with certainty) as its initial cash outlay (I_0) and its end-of-period cash flow ($\tilde{\Delta Y}$) has the following relationships with the rate of return on the Standard & Poor's 500-stock index ($\tilde{r}_M$):



* () represents the measure of probability

Consequently, we obtain the following information:

Scenario #	Description	Prob- ability	$\tilde{\Delta Y}$	$\tilde{r}_M$	$\tilde{e}$
1	good economy & striking oil	$\frac{1}{8}$	\$10M	15%	122%
2	good economy & striking no oil	$\frac{1}{8}$	\$ 2M	15%	-55%
3	bad economy & striking oil	$\frac{3}{8}$	\$ 8M	10%	78%
4	bad economy & striking no oil	$\frac{3}{8}$	\$ 0M	10%	-100%

Notice that all scenarios must be mutually exclusive and collectively exhaustive. Now it is easy to compute E_M , σ_M , $E(\tilde{\Delta Y})$, $\text{cov}(\tilde{\Delta Y}, \tilde{r}_M)$, $\text{cov}(\frac{\tilde{\Delta Y}}{E(\tilde{\Delta Y})}, \tilde{r}_M)$ and $\text{cov}(\tilde{e}, \tilde{r}_M)$, which are required by Equations (4.6), (4.11) and (4.12). In this case, we have

$$E_M = 0.1125$$

$$\sigma_M = 0.0216$$

$$E(\tilde{\Delta Y}) = 4.5$$

$$\text{cov}(\tilde{\Delta Y}, \tilde{r}_M) = -0.01875$$

$$\text{cov}(\frac{\tilde{\Delta Y}}{E(\tilde{\Delta Y})}, \tilde{r}_M) = -0.00417$$

$$\text{cov}(\tilde{e}, \tilde{r}_M) = -0.00417$$

Suppose the rate of return on the Treasury Bill, a surrogate for the riskless security, is estimated to be 5% for the relevant period. Then, the parameter $\phi = (E_M - r_f) / \sigma_M^2 = 133.96$. From Equation (4.6), we have

$$V_{\Delta Y} = \frac{4.5 - (133.96)(-0.01875)}{1 + 0.05} = 6.68 (M)$$

Or, from Equations (4.11), (4 .12), (4 .14), and (4 .16), we have

$$V_{\Delta Y} = \frac{4.5}{1 + (-0.3263)} = 6.68 (M)$$

Since $V_{\Delta Y} - I_0 = \$6.68M - \$5M = \$1.68M > 0$, managers should accept this project.

From section 1, we know $V_{\Delta Y} - I_0$ is also an appropriate index in selecting a representative alternative for a project from all the mutually exclusive alternatives of that project. Therefore, Equation (4.6), (4.11), or (4.12) can also be applied when evaluating an alternative.

SECTION 5

EXTENDING THE CAPM TO A MULTIPERIOD HORIZON

Suppose a security has $\tilde{Z} = [\tilde{Z}_1, \tilde{Z}_2, \dots, \tilde{Z}_\infty]$ as its prospect. (If the relevant horizon is only m periods, then $\tilde{Z}_t = 0$ for $t > m$.) How can we find V_Z through the CAPM?

Let V_{Z_1} = market value of $[\tilde{Z}_1, 0, \dots]$

V_{Z_2} = market value of $[0, \tilde{Z}_2, 0, \dots]$

and so on. The VAP says

$$V_Z = \sum_{t=1}^{\infty} V_{Z_t} \quad (5.1)$$

If we find V_{Z_t} for each t , we obtain V_Z immediately.

For each $[0, \dots, 0, \tilde{Z}_t, 0, \dots, 0]$ and each $\tau < t$, we define

$E_\tau(\tilde{Z}_t)$ = expected value of $\tilde{Z}_t$ at time τ and conditional on all information available at time τ .

V_τ = market value of $[0, \dots, 0, \tilde{Z}_t, 0, \dots, 0]$ at time τ .

$$\tilde{r}_{\tau+1} = \tilde{V}_{\tau+1}/V_\tau - 1$$

$E(\tilde{r}_{\tau+1})$ = expected value of $\tilde{r}_{\tau+1}$ at time τ .

$\tilde{e}_t$ = a random variable such that

$$\tilde{Z}_t = E_{t-1}(\tilde{Z}_t)(1 + \tilde{e}_t), \text{ where } E_{t-1}(\tilde{e}_t) = 0.$$

$\tilde{e}_\tau$ = a random variable such that

$$\tilde{E}_\tau(\tilde{Z}_t) = E_{\tau-1}(\tilde{Z}_t)(1 + \tilde{e}_\tau), \text{ where } E_{\tau-1}(\tilde{e}_\tau) = 0.$$

$\tilde{r}_{Mt}$ = rate of return on the market portfolio for period t .

$\tilde{r}_{M\tau}$ = rate of return on the market portfolio for period τ .

E_{Mt} = expected value of $\tilde{r}_{Mt}$

$E_{M\tau}$ = expected value of $\tilde{r}_{M\tau}$

σ_{Mt}^2 = variance of $\tilde{r}_{Mt}$

$\sigma_{M\tau}^2$ = variance of $\tilde{r}_{M\tau}$

r_{ft} = riskless rate of return for period t .

$r_{f\tau}$ = riskless rate of return for period τ .

$$\phi_t = (E_{Mt} - r_{ft})/\sigma_{Mt}^2$$

$$\phi_{\tau} = (E_{M\tau} - r_{f\tau}) / \sigma_{M\tau}^2$$

Also, let $E_{\tau-1}(\tilde{V}_{\tau})$ = expected value of $\tilde{V}_{\tau}$ at time $\tau-1$ and conditional on all information available at time $\tau-1$.

First, let us try to find V_{Z_t} , the market value of $[0, \dots, 0, \tilde{Z}_t, 0, \dots, 0]$. We are to apply the CAPM to period t , and then period $t-1$, and then period $t-2, \dots$, and then period 1.

Applying the CAPM to period t , we have

$$V_{t-1} = \frac{E_{t-1}(\tilde{Z}_t) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})]}{1 + r_{ft}} \quad (5.2)$$

or

$$V_{t-1} = \frac{E_{t-1}(\tilde{Z}_t)}{1 + E(\tilde{r}_t)} \quad (5.3)$$

where

$$E(\tilde{r}_t) = \frac{1 + r_{ft}}{1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})} - 1 \quad (5.4)$$

Fama [10] and Fama and MacBeth [13] have demonstrated that if the CAPM, as stated in Section 3, applies to each period, then it rules out relationships between uncertainty at $t-2$ about return

realized at $t - 1$ and uncertainty at $t - 2$ about the characteristics of the portfolio opportunity set (i.e., the subjective joint probability distribution of security returns realized at t) available at $t - 1$. If these relationships were existing, investors would have incentives to make their portfolio decisions at $t - 2$ to hedge against uncertainty about the characteristics of the portfolio opportunity set available at $t - 1$. The resulting pricing process is different from the pricing process in a CAPM setting. Therefore, in order to apply the CAPM to a multi-period horizon, we must assume that all the CAPM-relevant characteristics of the portfolio opportunity set available at time $t - 1$ must be known with certainty (nonstochastic) at time $t - 2$. These characteristics (original or derived) include r_{ft} , ϕ_t , $E(\tilde{r}_t)$, $\text{cov}(\tilde{e}_t, \tilde{r}_{Mt})$, $\text{cov}(\tilde{r}_t, \tilde{r}_{Mt})$, E_{Mt} , σ_{Mt}^2 , and λ_t .

Now suppose that we are at $t - 2$ and want to apply the CAPM to period $t - 1$. V_{t-1} becomes uncertain at $t - 2$; and from Equation (5.2) we know that the uncertainty of V_{t-1} comes from the uncertainty of $E_{t-1}(\tilde{Z}_t)$ only. As defined earlier,

$$\tilde{E}_{t-1}(\tilde{Z}_t) = E_{t-2}(\tilde{Z}_t)(1 + \tilde{e}_{t-1}) \quad (5.5)$$

and

$$E_{t-2}(\tilde{e}_{t-1}) = 0 \quad (5.6)$$

Then,

$$\begin{aligned}
\tilde{V}_{t-1} &= \frac{\tilde{E}_{t-1}(\tilde{Z}_t) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})]}{1 + r_{ft}} \\
&= \frac{E_{t-2}(\tilde{Z}_t)(1 + \tilde{e}_{t-1}) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})]}{1 + r_{ft}} \quad (5.7)
\end{aligned}$$

or

$$\tilde{V}_{t-1} = \frac{\tilde{E}_{t-1}(\tilde{Z}_t)}{1 + E(\tilde{r}_t)} = \frac{E_{t-2}(\tilde{Z}_t)(1 + \tilde{e}_{t-1})}{1 + E(\tilde{r}_t)} \quad (5.8)$$

where $E(\tilde{r}_t)$ is defined in Equation (5.4).

From Equation (5.5), (5.7) and (5.8), we know

$$E_{t-2}(\tilde{V}_{t-1}) = \frac{E_{t-2}(\tilde{Z}_t) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})]}{1 + r_{ft}} \quad (5.9)$$

$$= \frac{E_{t-2}(\tilde{Z}_t)}{1 + E(\tilde{r}_t)} \quad (5.10)$$

$$\text{cov}(\tilde{V}_{t-1}, \tilde{r}_{Mt-1}) = \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1}) \frac{E_{t-2}(\tilde{Z}_t)}{1 + E(\tilde{r}_t)} \quad (5.11)$$

and hence,

$$\frac{\text{cov}(\tilde{V}_{t-1}, \tilde{r}_{Mt-1})}{E_{t-2}(\tilde{V}_{t-1})} = \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1}) \quad (5.12)$$

Applying the CAPM to period $t - 1$, we have

$$\begin{aligned}
V_{t-2} &= \frac{E_{t-2}(\tilde{V}_{t-1}) - \phi_{t-1} \text{cov}(\tilde{V}_{t-1}, \tilde{r}_{Mt-1})}{1 + r_{ft-1}} \\
&= \frac{E_{t-2}(\tilde{V}_{t-1}) [1 - \phi_{t-1} \text{cov}(\tilde{V}_{t-1}, \tilde{r}_{Mt-1}) / E_{t-2}(\tilde{V}_{t-1})]}{1 + r_{ft-1}} \\
&= \frac{E_{t-2}(\tilde{V}_{t-1}) [1 - \phi_{t-1} \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1})]}{1 + r_{ft-1}} \quad (5.13)
\end{aligned}$$

or

$$V_{t-2} = \frac{E_{t-2}(\tilde{V}_{t-1})}{1 + E(\tilde{r}_{t-1})} \quad (5.14)$$

where

$$E(\tilde{r}_{t-1}) = \frac{1 + r_{ft-1}}{1 - \phi_{t-1} \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1})} - 1 \quad (5.15)$$

Putting Equation (5.9) into Equation (5.13), we have

$$\begin{aligned}
V_{t-2} &= \left\{ E_{t-2}(\tilde{Z}_t) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})] \right. \\
&\quad \left. [1 - \phi_{t-1} \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1})] \right\} / \left\{ (1 + r_{ft})(1 + r_{ft-1}) \right\} \\
&\quad (5.16)
\end{aligned}$$

Putting Equation (5.10) into Equation (5.14), we have

$$V_{t-2} = \frac{E_{t-2}(\tilde{Z}_t)}{[1 + E(\tilde{r}_t)][1 + E(\tilde{r}_{t-1})]} \quad (5.17)$$

Now suppose that we are at time $t - 3$ and want to apply the CAPM to period $t - 2$. V_{t-2} becomes uncertain at time $t - 3$. Again, all the CAPM-relevant characteristics of the portfolio opportunity set available at time $t - 2$ must be known with certainty (nonstochastic) at time $t - 3$. These characteristics include r_{ft} , r_{ft-1} , $E(\tilde{r}_t)$, $E(\tilde{r}_{t-1})$, $\text{cov}(\tilde{r}_t, \tilde{r}_{Mt})$, $\text{cov}(\tilde{r}_{t-1}, \tilde{r}_{Mt-1})$, E_{Mt} , E_{Mt-1} , σ_{Mt}^2 , σ_{Mt-1}^2 , λ_t , and λ_{t-1} . Hence, the uncertainty of V_{t-2} comes from the uncertainty of $E_{t-2}(\tilde{Z}_t)$ only. Similar procedure can be used to obtain V_{t-3} .

In general, for any $\tau < t$ we have

$$V_\tau = \left\{ E_\tau(\tilde{Z}_t) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})] [1 - \phi_{t-1} \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1})] \right. \\ \left. \dots \left[1 - \phi_{\tau+1} \text{cov}(\tilde{e}_{\tau+1}, \tilde{r}_{Mt+1}) \right] \right\} / \left[(1 + r_{ft}) \right. \\ \left. \dots (1 + r_{f\tau+1}) \right] \quad (.5.18)$$

or

$$V_\tau = \frac{E_\tau(\tilde{Z}_t)}{[1 + E(\tilde{r}_t)] [1 + E(\tilde{r}_{t-1})] \dots [1 + E(\tilde{r}_{\tau+1})]} \quad (5.19)$$

We are most interested in V_0 , which is the market value of $[0, 0, \dots, 0, \tilde{Z}_t, 0, \dots]$, i.e., $V_0 = V_{Z_t}$. The final result is that

$$\begin{aligned}
V_{Z_t} = & \left(E_0(\tilde{Z}_t) [1 - \phi_t \text{cov}(\tilde{e}_t, \tilde{r}_{Mt})] \right. \\
& [1 - \phi_{t-1} \text{cov}(\tilde{e}_{t-1}, \tilde{r}_{Mt-1})] \dots \\
& \left. [1 - \phi_1(\tilde{e}_1, \tilde{r}_{M1})] \right) / \left[(1 + r_{ft})(1 + r_{ft-1}) \dots \right. \\
& \left. (1 + r_{f1}) \right] \quad (5.20)
\end{aligned}$$

or

$$V_{Z_t} = \frac{E_0(\tilde{Z}_t)}{[1 + E(\tilde{r}_t)][1 + E(\tilde{r}_{t-1})] \dots [1 + E(\tilde{r}_1)]} \quad (5.21)$$

where

$$E(\tilde{r}_\tau) = \frac{1 + r_{f\tau}}{1 - \phi_\tau \text{cov}(\tilde{e}_\tau, \tilde{r}_{M\tau})} - 1 \quad (5.22)$$

for $\tau = 1, 2, \dots, t$.

Notice that in order to use Equations (5.20) through (5.22) properly, all the CAPM-relevant characteristics of portfolio opportunity sets available in period 1, period 2,... and period t must be assumed to be known with certainty (nonstochastic) at time 0. It is a quite restrictive assumption. For example, the expected rate of return in the last period, $E(\tilde{r}_t)$, must remain the same no matter what the state of nature turns out to be in the first $t - 1$ periods.

In Equation (5.20), the numerator is the certainty equivalent of $\tilde{Z}_t$ at time t and therefore is divided by a series of t riskless discount factors. In Equation (5.21), the denominator is the product of a series of t risk-adjusted discount factors and therefore is to divide the expected value of $\tilde{Z}_t$ at time 0.

Suppose we let

$$\alpha_t = [1 - \phi_t \text{ cov } (\tilde{e}_t, \tilde{r}_{Mt})][1 - \phi_{t-1} \text{ cov } (\tilde{e}_{t-1}, \tilde{r}_{Mt-1})] \dots [1 - \phi_1 \text{ cov } (\tilde{e}_1, \tilde{r}_{M1})] \quad (5.23)$$

Then, Equation (5.20) becomes

$$V_{Z_t} = \frac{E_0(\tilde{Z}_t) \alpha_t}{(1 + r_{ft})(1 + r_{ft-1}) \dots (1 + r_{f1})} \quad (5.24)$$

In a CAPM setting, α_t is a market-determined factor. It is not a consequence of the utility function of any manager or any other individual; it is a consequence of an equilibrium capital market. From the definition of α_t , we know α_t must be known with certainty at time 0.

To obtain the market value (V_Z) of $[\tilde{Z}_1, \tilde{Z}_2, \dots, \tilde{Z}_\infty]$, we simply use Equation (5.1). As a result,

$$V_Z = \sum_{t=1}^{\infty} \frac{E_0(\tilde{Z}_t) \alpha_t}{\prod_{\tau=1}^t (1 + r_{f\tau})} \quad (5.25)$$

or

$$V_Z = \sum_{t=1}^{\infty} \frac{E_0(\tilde{Z}_t)}{\prod_{\tau=1}^t [1 + E(\tilde{r}_{t\tau})]} \quad (5.26)$$

where

$$\alpha_t = [1 - \phi_t \text{cov}(\tilde{e}_{tt}, \tilde{r}_{Mt})][1 - \phi_{t-1} \text{cov}(\tilde{e}_{tt-1}, \tilde{r}_{Mt-1})] \dots [1 - \phi_1 \text{cov}(\tilde{e}_{t1}, \tilde{r}_{M1})] \quad (5.27)$$

$$E(\tilde{r}_{t\tau}) = \frac{\text{cov}(1 + r_{f\tau}, \tilde{e}_{t\tau})}{1 - \phi_{\tau} \text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau})} - 1 \quad (5.28)$$

Notice that, in $\tilde{r}_{t\tau}$ or $\tilde{e}_{t\tau}$, the "t" refers to the period $\tilde{Z}_t$ actually occurs and the "τ" refers to any period between 1 (included) and period t (included). Also notice that the following equalities do not have to hold:

$$\text{cov}(\tilde{e}_{t\tau_1}, \tilde{r}_{M\tau_1}) = \text{cov}(\tilde{e}_{t\tau_2}, \tilde{r}_{M\tau_2}) \quad (5.29)$$

$$\text{cov}(\tilde{e}_{t_1\tau}, \tilde{r}_{M\tau}) = \text{cov}(\tilde{e}_{t_2\tau}, \tilde{r}_{M\tau}) \quad (5.30)$$

$$E(\tilde{r}_{t\tau_1}) = E(\tilde{r}_{t\tau_2}) \quad (5.31)$$

$$E(\tilde{r}_{t_1\tau}) = E(\tilde{r}_{t_2\tau}) \quad (5.32)$$

Readers can find further discussions in [11].

Equation (5.25) is a complete form of the Certainty Equivalent (CE) model, while Equation (5.26) is a complete form of the Risk Adjusted Discount (RAD) model. Obviously, these two models are equivalent to each other.

In the following chapters, several techniques used in evaluating multiperiod projects are to be introduced. Since single-period projects are a special case of multiperiod projects, these techniques can be applied to single-period projects as well. To end this chapter, we emphasize that, in order to apply the CAPM to a multiperiod horizon, many market-determined factors, which are CAPM-relevant characteristics of the portfolio opportunity set available in each future period, must be assumed to be known with certainty (nonstochastic) at time 0. That is to say, once managers have got estimates of these factors, they must act as if these estimates were known with certainty (nonstochastic) in order to apply the CAPM properly. This assumption is restrictive; we just hope that the implications of the CAPM, when applied to a multiperiod horizon, are reasonably consistent with the observed phenomenon; and more importantly, the model can perform well in prediction.

SECTION 6

CERTAINTY EQUIVALENT MODEL IN MULTIPERIOD PROJECT EVALUATION: BASED ON THE CAPM

We reproduce Equations (5.25) and (5.27)

below:

$$V_2 = \sum_{t=1}^{\infty} \frac{E_0(\tilde{Z}_t) \alpha_t}{\prod_{\tau=1}^t (1 + r_{f\tau})} \quad (6.1)$$

$$\begin{aligned} \alpha_t = & [1 - \phi_t \text{ cov } (\tilde{e}_{tt}, \tilde{r}_{Mt})][1 - \phi_{t-1} \text{ cov } (\tilde{e}_{tt-1}, \tilde{r}_{Mt-1})] \\ & \dots [1 - \phi_1 \text{ cov } (\tilde{e}_{t1}, \tilde{r}_{M1})] \end{aligned} \quad (6.2)$$

Equations (6.1) and (6.2) are a complete form of the CE model in a CAPM setting. Now suppose a project has $[-I_0, \Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$ as its stream of cash flows. Let $[\tilde{Z}_1, \tilde{Z}_2, \dots, \tilde{Z}_\infty] = [\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$. Use Equations (6.1) and (6.2) to obtain $V_2 = V_{\Delta Y}$. Then, managers should accept this project if $V_{\Delta Y} - I_0 > 0$. The same procedure can be applied to mutually exclusive alternatives. In order to use Equation (6.1), managers must obtain all required data. Presented below is a scenario (state of nature) approach.

Suppose a two-period project needs \$8M (known with certainty) as its initial cash outlay (I_0). Its future cash flows, $\Delta\tilde{Y}_1$ and $\Delta\tilde{Y}_2$, and the rates of return on the Standard & Poor's 500-stock index for these two periods, $\tilde{r}_{M1}$ and $\tilde{r}_{M2}$, have the relationships described below.

Let $E(i, t)$ denote the event i which occurs in period t . Information at time 0 regarding $\Delta\tilde{Y}_1$ is given as follows:

	<u>Period 1</u>	<u>$\Delta\tilde{Y}_1$</u>	<u>$\tilde{e}_{11}$</u>	<u>$\tilde{r}_{M1}$</u>	<u>r_{f1}</u>
$(1/8)^*$	$E(1, 1)$	\$10M	122%	15%	5%
$(1/8)$	$E(2, 1)$	\$2M	-55%	15%	5%
$(3/8)$	$E(3, 1)$	\$8M	78%	10%	5%
$(3/8)$	$E(4, 1)$	\$0M	-100%	10%	5%

* () represents the measure of probability

Now,

$$V_{\Delta Y_1} = \frac{E_0(\Delta\tilde{Y}_1)\alpha_1}{1 + r_{f1}}$$

where

$$\alpha_1 = 1 - \phi_1 \text{ cov } (\tilde{e}_{11}, \tilde{r}_{M1})$$

After some calculation, we have

$$E_0(\Delta \tilde{Y}_1) = 4.5$$

$$\phi_1 = 133.96$$

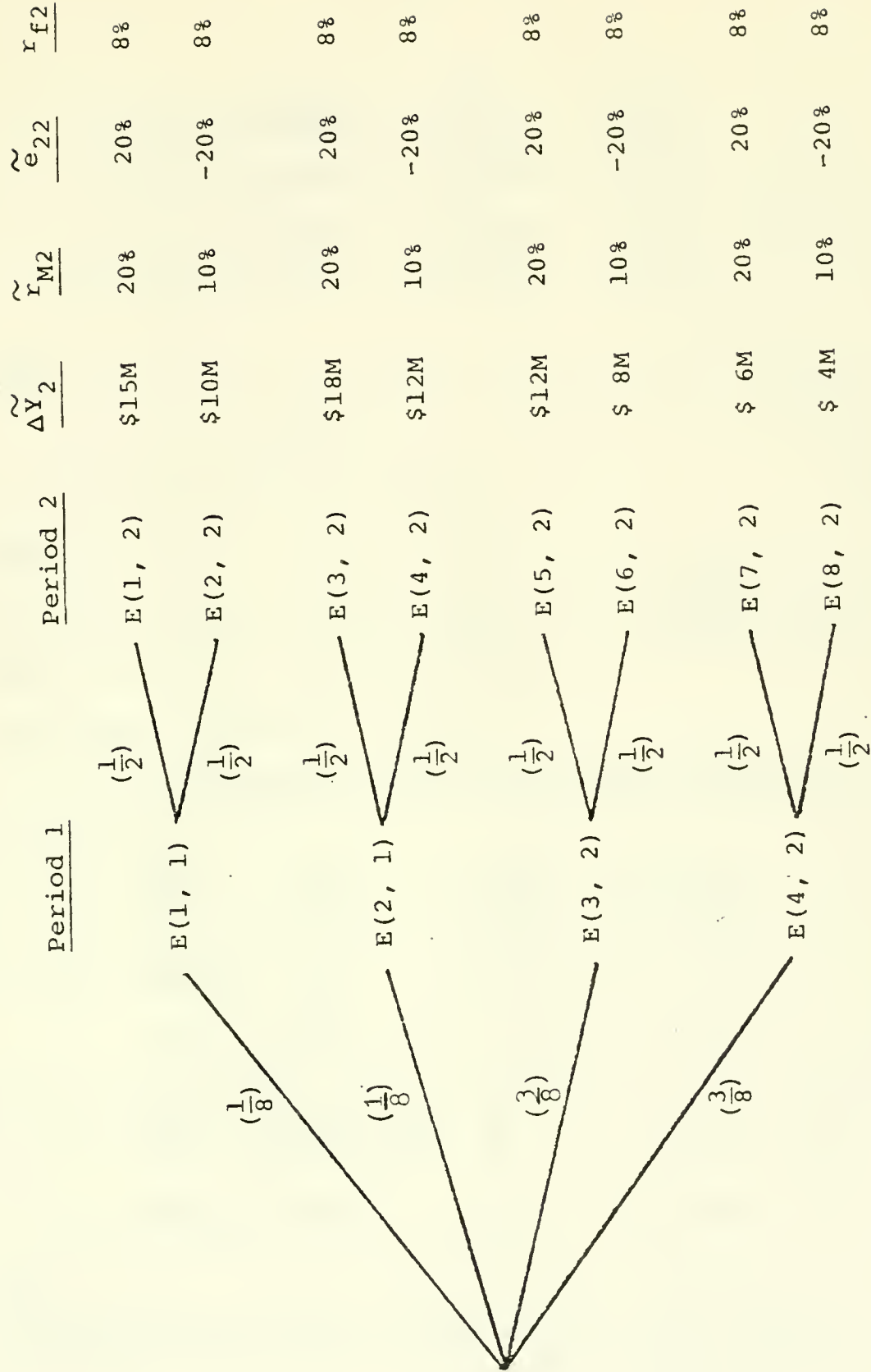
$$\text{cov}(\tilde{e}_{11}, \tilde{r}_{M1}) = -0.00417$$

$$\alpha_1 = 1.5586$$

and

$$V_{\Delta Y_1} = 6.68$$

Information at time 0 regarding $\Delta \tilde{Y}_2$ is given as follows:



Now,

$$\tilde{E}_1(\Delta\tilde{Y}_2) = \begin{cases} 12.5 & \text{if } E(1, 1) \text{ occurs in period 1} \\ 15 & \text{if } E(2, 1) \text{ occurs in period 2} \\ 10 & \text{if } E(3, 1) \text{ occurs in period 3} \\ 5 & \text{if } E(4, 1) \text{ occurs in period 4} \end{cases}$$

$$E_0(\Delta\tilde{Y}_2) = 12.5 \times \frac{1}{8} + 15 \times \frac{1}{8} + 10 \times \frac{3}{8} + 5 \times \frac{3}{8} = 9.0625$$

Then,

$$\tilde{e}_{21} = \frac{\tilde{E}_1(\Delta\tilde{Y}_2)}{E_0(\Delta\tilde{Y}_2)} - 1 = \begin{cases} 0.379 & \text{if } E(1, 1) \text{ occurs in period 1} \\ 0.655 & \text{if } E(2, 1) \text{ occurs in period 2} \\ 0.103 & \text{if } E(3, 1) \text{ occurs in period 3} \\ -0.448 & \text{if } E(4, 1) \text{ occurs in period 4} \end{cases}$$

And, $\tilde{e}_{21}$ and $\tilde{r}_{M1}$ have the following association:

<u>Period 1</u>	<u>Prob.</u>	<u>$\tilde{e}_{21}$</u>	<u>$\tilde{r}_{M1}$</u>
E(1, 1)	$\frac{1}{8}$	0.379	15%
E(2, 1)	$\frac{1}{8}$	0.655	15%
E(3, 1)	$\frac{3}{8}$	0.103	10%
E(4, 1)	$\frac{3}{8}$	-0.448	10%

And, no matter what event occurs in period 1, $\tilde{e}_{22}$ and $\tilde{r}_{M2}$ have the following association:

<u>Prob.</u>	<u>$\tilde{e}_{22}$</u>	<u>$\tilde{r}_{M2}$</u>
$\frac{1}{2}$	0.2	20%
$\frac{1}{2}$	0.2	10%

Now,

$$V_{\Delta Y_2} = \frac{E_0(\Delta \tilde{Y}_2) \alpha_2}{(1 + r_{f1})(1 + r_{f2})}$$

where

$$\alpha_2 = [1 - \phi_1 \text{ cov } (\tilde{e}_{21}, \tilde{r}_{M1})][1 - \phi_2 \text{ cov } (\tilde{e}_{22}, \tilde{r}_{M2})]$$

After some calculation, we have

$$E_0(\Delta \tilde{Y}_2) = 9.0625$$

$$\phi_2 = 28$$

$$\phi_1 = 133.96$$

$$\text{cov } (\tilde{e}_{22}, \tilde{r}_{M2}) = 0.01$$

$$\text{cov } (\tilde{e}_{21}, \tilde{r}_{M1}) = 0.00645$$

$$\alpha_2 = 0.0979$$

and

$$V_{\Delta Y_2} = 0.78$$

Therefore,

$$V_{\Delta Y} = V_{\Delta Y_1} + V_{\Delta Y_2} = 6.68 + 0.78 = 7.46 \text{ (M)}$$

Since $V_{\Delta Y} - I_0 = 7.46 - 8 = -0.54 < 0$, managers should not accept the project.

In the preceding scenario structure, $\Delta \tilde{Y}_1$ has four mutually exclusive and collectively exhaustive scenarios (namely, $E(1, 1)$, $E(2, 1)$, $E(3, 1)$ and $E(4, 1)$) and $\Delta \tilde{Y}_2$ has eight mutually exclusive and collectively exhaustive scenarios (namely, $E(1, 1) + E(1, 2)$, $E(1, 1) + E(2, 2)$, $E(2, 1) + E(3, 2)$, $E(2, 1) + E(4, 2)$, $E(3, 1) + E(5, 2)$, $E(3, 1) + E(6, 2)$, $E(4, 1) + E(7, 2)$, and $E(4, 1) + E(8, 2)$.) Notice that, in constructing scenarios for $\Delta \tilde{Y}_2$, we do not have to use $E(1, 1)$, $E(2, 1)$, $E(3, 1)$ and $E(4, 1)$ in period 1; we may want to construct other events in period 1 for $\Delta \tilde{Y}_2$. Also notice that, in our artificial example, data are purposely devised so that E_{M2} , σ_{M2}^2 , r_{f2} , $\text{cov}(\tilde{e}_{22}, \tilde{r}_{M2})$, $E_1(\tilde{r}_{22})$, $\text{cov}(\tilde{r}_{22}, \tilde{r}_{M2})$, ϕ_2 , and λ_{22} remain unchanged no matter what event occurs in period 1. This may never happen in the real world, but it is what the CAPM assumes in a multiperiod horizon.

Sometimes we may wish to assume

$$\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) = c_\tau \quad \text{for all } t \geq \tau, \quad \tau = 1, 2, 3, \dots \quad (16.3)$$

Equation (6.3) says that for any period τ , $\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau})$ is unchanged across the cash flows $\Delta\tilde{Y}_t$, $t=1, 2, 3, \dots$. Since all the cash flows belong to the same prospect initiated by I_0 , it may not ^{be} unreasonable to assume that the reassessment in period τ of cash flow $\Delta\tilde{Y}_{t_1}$ covaries with the market return in period τ in the same way as the reassessment in period τ of any other cash flow $\Delta\tilde{Y}_{t_2}$ covaries with the market return in period τ . Then,

$$\alpha_t = \alpha_{t-1}(1 - \phi_t c_t) \quad \text{for } t \geq 2 \quad (6.4)$$

and

$$\alpha_1 = 1 - \phi_1 c_1 \quad (6.5)$$

Equations (6.4) and (6.5) imply

- (1) If $0 < 1 - \phi_t c_t < 1$, then $\alpha_t < \alpha_{t-1}$ and the capital market prefers the risk structure of $\Delta\tilde{Y}_{t-1}$ to that of $\Delta\tilde{Y}_t$. To see this, let $V_{\Delta Y_{t-1}}$ and $V_{\Delta Y_t}$ be the market values of $[0, 0, \dots, 0, \Delta\tilde{Y}_{t-1}, 0, \dots, 0]$ and $[0, 0, \dots, 0, \Delta\tilde{Y}_t, 0, \dots, 0]$, respectively. Then,

$$V_{\Delta Y_{t-1}} = E_0(\Delta\tilde{Y}_{t-1}) \frac{\alpha_{t-1}}{\prod_{\tau=1}^{t-1} (1 + r_{f\tau})}$$

and

$$V_{\Delta Y_t} = E_0(\Delta \tilde{Y}_t) \frac{\alpha_t}{\prod_{\gamma=1}^t (1 + r_{f\gamma})} \quad \dots$$

$$= E_0(\Delta \tilde{Y}_t) \left[\frac{\alpha_{t-1}}{\prod_{\gamma=1}^{t-1} (1 + r_{f\gamma})} \right] \left[\frac{1 - \phi_t c_t}{1 + r_{ft}} \right]$$

To derive the market value from the expected cash flow, $E_0(\Delta \tilde{Y}_t)$ is multiplied by one extra term as compared to $E_0(\Delta \tilde{Y}_{t-1})$. This extra term is $(1 - \phi_t c_t)/(1 + r_{ft})$. The denominator, $1 + r_{ft}$, takes care of the incremental elapse of time involved in $\Delta \tilde{Y}_t$ as compared to $\Delta \tilde{Y}_{t-1}$. The numerator, $1 - \phi_t c_t$, takes care of the incremental risk involved in $\Delta \tilde{Y}_t$ as compared to $\Delta \tilde{Y}_{t-1}$. Now that $0 < 1 - \phi_t c_t < 1$, $\Delta \tilde{Y}_t$ must be more risky than $\Delta \tilde{Y}_{t-1}$ as perceived by the capital market.

- (2) If $1 - \phi_t c_t > 1$, then $\alpha_t > \alpha_{t-1}$ and the capital market prefers the risk structure of $\Delta \tilde{Y}_t$ to that of $\Delta \tilde{Y}_{t-1}$.
- (3) If $1 - \phi_t c_t = 1$, then $\alpha_t = \alpha_{t-1}$ and the risk structure of $\Delta \tilde{Y}_t$ and that of $\Delta \tilde{Y}_{t-1}$ make no difference to the capital market.

In addition to (16.3), we may wish to assume the following simplifying assumptions:

$$\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) \text{ is the same for all } \tau \leq t, \quad t = 1, 2, 3, \dots \quad (6.6)$$

and,

$$\phi_t = \phi \quad \text{for all } t \quad (6.7)$$

Equation (6.6) says that for any cash flow $\Delta\tilde{Y}_t$, $\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau})$ is the same through time τ . Since $\tilde{e}_{t1}, \tilde{e}_{t2}, \dots$ are all related to the same cash flow $\Delta\tilde{Y}_t$, it may not be unreasonable to assume that the reassessment in period τ_1 of cash flow $\Delta\tilde{Y}_t$ covaries with the market return in period τ_1 in the same way as the reassessment in period τ_2 of the same cash flow $\Delta\tilde{Y}_t$ covaries with the market return in period τ_2 . Equation (6.7) says that the risk price, $\phi_t = (E_{Mt} - r_{ft})/\sigma_{Mt}^2$, is the same through time. It may or may not be reasonable, depending upon the future market situation we may predict.

Anyway, under Equation (6.3) and Equation (6.6),

$$\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) = c \text{ for all } t \text{ and } \tau \quad (6.8)$$

Hence, under Equation (6.7), we have

$$\alpha_t = (1 - \phi c)^t \quad \text{for all } t \quad (6.9)$$

Equation (6.9) implies

- (1) If $0 < 1 - \phi c < 1$, then α_t decreases with time and the capital market prefers the risk structure of $\Delta \tilde{Y}_t$ to that of any subsequent cash flow.
- (2) If $1 - \phi c > 1$, then α_t increases with time and the capital market prefers the risk structure of $\Delta \tilde{Y}_t$ to that of any preceding cash flow.
- (3) If $1 - \phi c = 1$, then $\alpha_t = 1$ for all t and $[\Delta \tilde{Y}_1, \Delta \tilde{Y}_2, \dots, \Delta \tilde{Y}_\infty]$ is actually riskless.

When managers make assumptions (6.3), (6.6) and (6.7) (or make assumption (6.3) alone), they must realize what their assumptions imply. If, according to managers' past experience, future expectation; and other source of information these implications are not likely to be observed in the capital market, managers had better drop these assumptions.

To end this chapter, we emphasize again that α_t and r_{ft} are market-determined factors. In particular, α_t is not derived from the utility function of any individual.

When managers try to predict α_t and r_{ft} , they must think on behalf of the capital market as a whole. Once they have got estimates of α_t and r_{ft} , they must work with these estimates as if these estimates were known with certainty in order to use Equation (6.1).

SECTION 7

RISK ADJUSTED DISCOUNT MODEL IN MULTIPERIOD PROJECT EVALUATION: BASED ON THE CAPM

We reproduce Equations (5.26) and (5.28)

below:

$$V_Z = \sum_{t=1}^{\infty} \frac{E_0(\tilde{Z}_t)}{\prod_{\tau=1}^t [1 + E(\tilde{r}_{t\tau})]} \quad (7.1)$$

$$E(\tilde{r}_{t\tau}) = \frac{1 + r_{f\tau}}{1 - \phi_{\tau} \text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau})} - 1 \quad (7.2)$$

Equations (7.1) and (7.2) are a complete form of the RAD model in a CAPM setting. Now suppose a project has $[-I_0, \Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_{\infty}]$ as its stream of cash flows. Let $[\tilde{Z}_1, \tilde{Z}_2, \dots, \tilde{Z}_{\infty}] = [\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_{\infty}]$. Use Equations (7.1) and (7.2) to obtain $V_Z = V_{\Delta Y}$. Then, managers should accept this project if $V_{\Delta Y} - I_0 > 0$. The same procedure can be applied to mutually exclusive alternatives. As mentioned earlier, all market-determined factors $r_{f\tau}$, ϕ_{τ} , $\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau})$, and $E(\tilde{r}_{t\tau})$, where $\tau \leq t$, and $t = 1, 2, \dots, \infty$, must be known with

certainty at time 0 in order to use the CAPM properly.

Managers are faced with the same kind of problem in this model as in the CE model. The example cited in the previous section can also be an example in the RAD model.

Managers may wish to make some simplifying assumptions. As in the case of the CE model, managers should take into consideration the implications of their assumptions before they put their assumptions into work.

Sometimes managers may wish to assume

$$\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) = c_\tau \quad \text{for all } t \geq \tau, \quad \tau = 1, 2, 3, \dots \quad (7.3)$$

Then,

$$E(\tilde{r}_{t\tau}) = E(\tilde{r}_\tau) = \frac{1 + r_{f\tau}}{1 - \phi_\tau c_\tau} - 1 \quad \text{for all } t \geq \tau, \quad \tau = 1, 2, 3, \dots \quad (7.4)$$

and

$$\prod_{\tau=1}^t [1 + E(\tilde{r}_{t\tau})] = \prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] \quad (7.5)$$

Equation (7.4) says that for each period τ , the expected rate of return remains unchanged across the cash flows $\Delta\tilde{Y}_t$, for $t = 1, 2, \dots$. Then,

$$V_{\Delta Y} = \sum_{t=1}^{\infty} \frac{\frac{E_0(\Delta\tilde{Y}_t)}{t}}{\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)]} \quad (7.6)$$

$$\begin{aligned}
&= \frac{E_0(\Delta\tilde{Y}_1)}{[1 + E(\tilde{r}_1)]} + \frac{E_0(\Delta\tilde{Y}_2)}{[1 + E(\tilde{r}_1)][1 + E(\tilde{r}_2)]} \\
&\quad + \frac{E_0(\Delta\tilde{Y}_3)}{[1 + E(\tilde{r}_1)][1 + E(\tilde{r}_2)][1 + E(\tilde{r}_3)]} + \dots\dots\dots
\end{aligned}
\tag{7.7}$$

Equations (7.4) and (7.5) imply

(1) If $0 < 1 - \phi_t c_t < 1$, then

$$1 + E(\tilde{r}_t) > 1 + r_{ft} \quad \text{and}$$

$$\begin{aligned}
\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] &= [1 + E(\tilde{r}_t)] \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)] \\
&> (1 + r_{ft}) \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)]
\end{aligned}
\tag{7.8}$$

Inequality (7.8) says that the capital market prefers the risk structure of $\Delta\tilde{Y}_{t-1}$ to that of $\Delta\tilde{Y}_t$.

(2) If $1 - \phi_t c_t > 1$, then

$$1 + E(\tilde{r}_t) < 1 + r_{ft} \quad \text{and}$$

$$\begin{aligned}
\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] &= [1 + E(\tilde{r}_t)] \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)] \\
&< (1 + r_{ft}) \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)]
\end{aligned}
\tag{7.9}$$

Inequality (7.9) says that the capital market prefers the risk structure of $\Delta \tilde{Y}_t$ to that of $\Delta \tilde{Y}_{t-1}$.

- (3) If $1 - \phi_t c_t = 1$, then
 $1 + E(\tilde{r}_t) = 1 + r_{ft}$ and

$$\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] = (1 + r_{ft}) \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)] \quad (7.10)$$

Equation (7.10) says that the risk structure of $\Delta \tilde{Y}_t$ and that of $\Delta \tilde{Y}_{t-1}$ make no difference to the capital market.

The implications obtained here are exactly the same as those obtained in the CE model.

In addition to Equation (7.3), managers may wish to make the following simplifying assumptions:

$$\text{cov} (\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) \text{ is the same for all } \tau \leq t, \quad t = 1, 2, 3, \dots \quad (7.11)$$

and,

$$\phi_t = \phi \quad \text{for all } t \quad (7.12)$$

Then, under Equations (7.3) and (7.11),

$$\text{cov} (\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) = c \quad \text{for all } t \text{ and } \tau \quad (7.13)$$

Hence, under Equations (7.12) and (7.13),

$$E(\tilde{r}_{t\tau}) = E(\tilde{r}_\tau) = \frac{1 + r_{f\tau}}{1 - \phi c} - 1 \quad \text{for all } t \text{ and all } \tau \leq t \quad (7.14)$$

and

$$\prod_{\tau=1}^t [1 + E(\tilde{r}_{t\tau})] = \prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] \quad (7.15)$$

Then,

$$V_{\Delta Y} = \sum_{t=1}^{\infty} \frac{E_0(\Delta \tilde{Y}_t)}{\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)]} \quad (7.16)$$

$$\begin{aligned} &= \frac{E_0(\Delta \tilde{Y}_1)}{[1 + E(\tilde{r}_1)]} + \frac{E_0(\Delta \tilde{Y}_2)}{[1 + E(\tilde{r}_1)][1 + E(\tilde{r}_2)]} \\ &\quad + \frac{E_0(\Delta \tilde{Y}_3)}{[1 + E(\tilde{r}_1)][1 + E(\tilde{r}_2)][1 + E(\tilde{r}_3)]} + \dots \quad (7.17) \end{aligned}$$

Equations (7.14) and (7.15) imply

(1) If $0 < 1 - \phi c < 1$, then

$$1 + E(\tilde{r}_t) > 1 + r_{ft} \quad \text{for all } t \text{ and}$$

$$\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] > (1 + r_{ft}) \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)] \quad (7.18)$$

for any $t \geq 2$. Inequality (7.18) says that the capital market prefers the risk structure

of $\Delta\tilde{Y}_t$ to that of any subsequent cash flow.

(2) If $1 - \phi c > 1$, then

$1 + E(\tilde{r}_t) < 1 + r_{ft}$ for all t and

$$\prod_{\tau=1}^t [1 + E(\tilde{r}_\tau)] < (1 + r_{ft}) \prod_{\tau=1}^{t-1} [1 + E(\tilde{r}_\tau)] \quad (7.19)$$

for any $t \geq 2$. Inequality (7.19) says that the capital market prefers the risk structure of $\Delta\tilde{Y}_t$ to that of any preceding cash flow.

(3) If $1 - \phi c = 1$, then

$1 + E(\tilde{r}_t) = 1 + r_{ft}$ for all t and

$[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$ is actually riskless.

Again, the implications obtained here are exactly the same as those obtained in the CE model.

Notice that the CE model and the RAD model are equivalent to each other. . If the same set of restrictions is imposed on both the CE model and the RAD model, the same implication is expected for both models.

The final remark made in this chapter is that $E(\tilde{r}_{t\tau})$ is also a market-determined factor. When managers try to predict $E(\tilde{r}_{t\tau})$, they must think on behalf of the capital market as a whole. Once they have got the estimate of $E(\tilde{r}_{t\tau})$, they must act as if the estimate were known with certainty in order to use Equation (7.1).

SECTION 8

COST-OF-CAPITAL MODEL IN MULTIPERIOD PROJECT EVALUATION:

BASED ON THE CAPM

Let $[-I_0, \Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$ be the stream of cash flows of a project. Under 3 assumptions outlined below, the CE model, i.e., Equations (5.25) and (5.27), can be reduced to

$$V_{\Delta Y} = \sum_{t=1}^{\infty} \frac{E_0(\Delta\tilde{Y}_t)(1 - \phi c)^t}{(1 + r_f)^t} \quad (8.1)$$

and the RAD model, i.e., Equations (5.26) and (5.28), can be reduced to

$$V_{\Delta Y} = \sum_{t=1}^{\infty} \frac{E_0(\Delta\tilde{Y}_t)}{[1 + E(\tilde{r})]^t} \quad (8.2)$$

$$\text{where } E(\tilde{r}) = \frac{1 + r_f}{1 - \phi c} - 1 \quad (8.3)$$

Obviously, Equation (8.1) and Equation (8.2) are equivalent. We shall choose Equation (8.2) to work with. Equation (8.2) is usually called the Cost-of-Capital model. And $E(\tilde{r})$ in Equation (8.2) is called the cost of capital.

The 3 assumptions needed here are

$$(1) \quad \text{cov} (\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) = c \quad \text{for all } t \text{ and } \tau \quad (8.4)$$

$$(2) \quad \phi_t = \phi \quad \text{for all } t \quad (8.5)$$

$$(3) \quad r_{ft} = r_f \quad \text{for all } t \quad (8.6)$$

Having gone through Section 6 and Section 7, readers can derive Equation (8.1) and Equation (8.2) immediately, based on (8.4), (8.5), and (8.6).

The Cost-of-Capital model has the following implications:

- (1) If $0 < 1 - \phi c < 1$, then $E(\tilde{r}) > r_f$ and $[1 + E(\tilde{r})]^t > [1 + E(\tilde{r})]^{t-1}(1 + r_f)$ for any $t \geq 1$. The capital market prefers the risk structure of $\Delta \tilde{Y}_t$ to that of any subsequent cash flow.
- (2) If $1 - \phi c > 1$, then $E(\tilde{r}) < r_f$ and $[1 + E(\tilde{r})]^t < [1 + E(\tilde{r})]^{t-1}(1 + r_f)$ for any $t \geq 1$. The capital market prefers the risk structure of $\Delta \tilde{Y}_t$ to that of any preceding cash flow.
- (3) If $1 - \phi c = 1$, then $E(\tilde{r}) = r_f$ and $[1 + E(\tilde{r})]^t = (1 + r_f)^t$ for any $t \geq 1$. $[\Delta \tilde{Y}_1, \Delta \tilde{Y}_2, \dots]$ is actually riskless.

In almost all cases where the Cost-of-Capital model is used, the single risk-adjusted-discount rate, $E(\tilde{r})$, is chosen to be greater than the single riskless rate of return, r_f .

However, given that $E(\tilde{r}) > r_f$, any cash flow $\Delta\tilde{Y}_t$ must be less risky than its subsequent cash flows. That is to say, given that $E(\tilde{r}) > r_f$, the Cost-of-Capital model can not correctly evaluate a prospect $[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots]$ unless $\Delta\tilde{Y}_1$ is less risky than $\Delta\tilde{Y}_2$, $\Delta\tilde{Y}_2$ is less risky than $\Delta\tilde{Y}_3$, and so on. In other words, given a prospect $[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots]$ such that all cash flows are equally risky, the Cost-of-Capital model with $E(\tilde{r}) > r_f$ is not an appropriate model to use; or, given a prospect $[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots]$ such that $\Delta\tilde{Y}_1$ is more risky than $\Delta\tilde{Y}_2$, $\Delta\tilde{Y}_2$ is less risky than $\Delta\tilde{Y}_3$, $\Delta\tilde{Y}_3$ is more risky than $\Delta\tilde{Y}_4$, and so on, the Cost-of-Capital model with $E(\tilde{r}) > r_f$ is not an appropriate model to use either. This observation was first made by Robichek and Myers [31].

In some rare situations, the Cost-of-Capital model is employed with $E(\tilde{r}) < r_f$ or $E(\tilde{r}) = r_f$. The Cost-of-Capital model with $E(\tilde{r}) < r_f$ can not correctly evaluate a prospect $[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots]$ unless $\Delta\tilde{Y}_1$ is more risky than $\Delta\tilde{Y}_2$, $\Delta\tilde{Y}_2$ is more risky than $\Delta\tilde{Y}_3$, and so on. The Cost-of-capital model with $E(\tilde{r}) = r_f$ can not correctly evaluate a prospect $[\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots]$ unless all cash flows are riskless.

Therefore, when managers make the 3 assumptions (8.4), (8.5), and (8.6) in order to use the Cost-of-Capital model, they must understand the consequent implications of these 3

assumptions. If these implications are not likely to be observed in the capital market, managers had better relax some or all of these 3 assumptions and go back to earlier models in which fewer assumptions are made.

To further simplify the Cost-of-Capital model, we may assume

$$\lambda_{t\tau} = \lambda \quad \text{for all } t \text{ and } \tau \quad (8.7)$$

In its original form,

$$\lambda_{t\tau} = \frac{\text{cov}(\tilde{r}_{t\tau}, \tilde{r}_{M\tau})}{\sigma_{M\tau}^2} \quad \text{for all } t \text{ and } \tau \quad (8.8)$$

Also,

$$E(\tilde{r}_{t\tau}) = r_{f\tau} + \phi_{\tau} \text{cov}(\tilde{r}_{t\tau}, \tilde{r}_{M\tau}) \quad (8.9)$$

$$= \frac{1 + r_{f\tau}}{1 - \phi_{\tau} \text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau})} \quad \text{for all } t \text{ and } \tau \quad (8.10)$$

Now, $\text{cov}(\tilde{e}_{t\tau}, \tilde{r}_{M\tau}) = c$, $\phi_t = \phi$, and $r_{ft} = r_f$ for all t and τ , based on (8.4), (8.5), and (8.6). Then, from Equations (8.9) and (8.10), $\text{cov}(\tilde{r}_{t\tau}, \tilde{r}_{M\tau})$ must be

a constant for all t and τ . Then, $\lambda_{t\tau} = \lambda$ for all t and τ if and only if $\sigma_{M\tau}^2$ is a constant for all τ , according to Equation (8.8). But, $\sigma_{M\tau}^2$ is a constant for all τ if and only if $E_{M\tau}$ is a constant for all τ , since

$\phi_\tau = (E_{M\tau} - r_{f\tau})/\sigma_{M\tau}^2$ and both ϕ_τ and $r_{f\tau}$ are assumed to be a constant for all τ . Therefore, the Cost-of-Capital model together with the assumption of Equation (8.7) imply that E_{Mt} , σ_{Mt}^2 , r_{ft} and ϕ_t are all unchanged over time.

To obtain the $E(\tilde{r})$ in Equation (8.2), managers must estimate r_f , ϕ , and c for future periods. Usually, managers look back to historical data to form a rough estimate. This estimate is then adjusted for other information concerning the future. Hopefully, the final estimate is unbiased about the future, given all currently available information.

Suppose a similar project has been undertaken before, either by the same firm or by another firm, and the relevant data are available. Then, those data are potentially useful for estimating $E(\tilde{r})$ of the current project. The following example is found in Chapter 7 of [38]. Let a new machine have the following stream of expected cash flows

$-I_0$	$E(\Delta\tilde{Y}_1)$	$E(\Delta\tilde{Y}_2)$	$E(\Delta\tilde{Y}_3)$	$E(\Delta\tilde{Y}_4)$	$E(\Delta\tilde{Y}_5)$
-130	35	32	28	25	22

$E(\Delta\tilde{Y}_6)$	$E(\Delta\tilde{Y}_7)$	$E(\Delta\tilde{Y}_8)$	$E(\Delta\tilde{Y}_9)$	$(E\Delta\tilde{Y}_{10})$
18	15	12	10	8

A similar machine, bought earlier, has the historical data as given in the Table 8.1 .

(1) Time	(2) Actual Cash Flow	(3) Secun- dary Market Value	(4) Machine's Rate of Return*	(5) S&P's Return	(6) Riskless Rate of Return
0	-110	110			
1	20	100	0.091	0.09	0.03
2	27	84	0.110	0.15	0.05
3	22	64	0.024	(0.04)	0.04
4	26	53	0.234	0.23	0.06
5	20	40	0.132	0.16	0.04
6	18	33	0.275	0.26	0.05
7	13	24	0.121	(0.10)	0.05
8	12	19	0.292	0.25	0.07
9	8	13	0.105	0.12	0.05
10	5	9	0.077	(0.02)	0.06

$$* (4)_t = [(2)_t + (3)_t - (3)_{t-1}] / (3)_{t-1}$$

Using Equation (3.6) as the ordinary-least-square regression equation, we obtain $\lambda = 0.53$.

TABLE 8.1

Suppose that managers believe the Cost-of-Capital model together with the assumption of Equation (8.7) will perform well in getting the reality of the new machine. Also suppose that managers believe the λ of the old but similar machine is a good proxy for that of the new machine, and, for each of the next ten periods,

riskless rate of return = 0.05

expected rate of return on S&P's 500-stock index = 0.11.

Then, managers can use Equation (8.2) with $E(\tilde{r}) = 0.05 + (0.11 - 0.05)0.53 = 0.082$. After some calculation, $V_{\Delta Y} = 150$. Since $V_{\Delta Y} - I_0 = 150 - 130 = 20 > 0$, managers should buy the new machine.

The problems with this example include

- (1) Does column (4) of Table (8.1) really measure the rate of return for the old machine? The machine's secondary market is a product market, which in most cases is not ^a perfect, efficient market. Then the market value established in this product market is not the same term we have talked about in a capital market. Also, the original cash outlay ($I_0 = 110$) is also a market price in a product market, not in a capital market. As a result, column (4) of Table (8.1)

does not really measure the rate of return in the CAPM setting.

- (2) Does it violate the assumption of Equation (8.6) that column (6) of Table (8.1) does not have a constant? Strictly speaking, it does. However, the variation of the riskless rate of return is not too serious. As a result, it may not cause as much trouble as the previous problem.

For some one-project firms whose common shares are publicly traded in a perfect efficient market, the historical λ of the common shares may be used in evaluating similar projects without having the first problem mentioned above.

Often the so-called weighted average cost of capital is used to evaluate a project. The weighted average cost of capital is the weighted average of the expected rates of return on bonds and shares. The evaluation formula involving the weighted average cost of capital is quite different from Equation (8.2) and invokes many additional assumptions. In the following, we shall develop this particular formula under the multiperiod setting of the CAPM.

Let $[-I_0, \Delta\tilde{Y}]$ is the stream of cash flows of a project, where I_0 is the initial cash outlay and $\Delta\tilde{Y} = [\Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots, \Delta\tilde{Y}_\infty]$ is the prospect. Let $\tilde{Y}' = [\tilde{Y}'_1, \tilde{Y}'_2, \dots, \tilde{Y}'_\infty]$

be the stream of future cash flows to all current security holders if the project is not adopted. Let $\tilde{R}' = [\tilde{R}'_1, \tilde{R}'_2, \dots, \tilde{R}'_\infty]$ be the stream of future cash flows to current bondholders if the project is not adopted. Let $\tilde{D}' = [\tilde{D}'_1, \tilde{D}'_2, \dots, \tilde{D}'_\infty]$ be the stream of future cash flows to current shareholders if the project is not adopted. Let $\tilde{Y} = [\tilde{Y}_1, \tilde{Y}_2, \dots, \tilde{Y}_\infty]$ be the stream of future cash flows to all current and new security holders if the project is adopted. Let $\tilde{R} = [\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_\infty]$ be the stream of future cash flows to all current and new bondholders if the project is adopted. Let $\tilde{D} = [\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_\infty]$ be the stream of future cash flows to all current and new shareholders if the project is adopted.

Assume that $\tilde{R}'$ and $\tilde{D}'$ satisfy the assumptions made in the Cost-of-Capital model, i.e., Equations (8.4) through (8.6). Furthermore, we assume

$$E_0(\tilde{R}'_t) = \bar{R}' = \text{constant} \quad \text{for } 1 \leq t \leq \infty \quad (8.11)$$

$$E_0(\tilde{D}'_t) = \bar{D}' = \text{constant} \quad \text{for } 1 \leq t \leq \infty \quad (8.12)$$

Equation (8.11) says that the time-0 expectations of all the future bond payments are identical with each other. Equation (8.12) says that the time-0 expectations of all the future dividend payments are identical with each other. Both equations are referred to the situation in which the project

is not adopted. Then,

$$B' = \sum_{t=1}^{\infty} \frac{\bar{R}'}{(1 + K'_B)^t} = \frac{\bar{R}'}{K'_B} \quad (8.13)$$

$$S' = \sum_{t=1}^{\infty} \frac{\bar{D}'}{(1 + K'_S)^t} = \frac{\bar{D}'}{K'_S} \quad (8.14)$$

where B' = market value of current bonds
if the project is not adopted
 S' = market value of current shares
if the project is not adopted
 K'_B = cost of capital of $\tilde{R}'$
 K'_S = cost of capital of $\tilde{D}'$

Let $\tilde{X}'_t$, $\tilde{DP}'_t$, and $\tilde{I}'_t$ be the cash earnings before interest and tax expenses, the depreciation expenses, and the net investment (=investment - disinvestment), respectively, in period t , if the project is not adopted. Equations (1.1) and (1.2) tell us that

$$\tilde{Y}' = \tilde{R}' + \tilde{D}' = \left[(1 - g)\tilde{X}' + g\tilde{DP}' - \tilde{I}' \right] + g\tilde{R}'$$

where g = tax rate. Let

$$\tilde{W}' = (1 - g)\tilde{X}' + g\tilde{DP}' - \tilde{I}'$$

We can interpret $\tilde{W}'$ as the stream of future cash flows to all current shareholders of the unlevered (i.e., all-equity) firm if the project is not adopted. Then,

$$\tilde{Y}' = \tilde{R}' + \tilde{D}' = \tilde{W}' + g \tilde{R}' \quad (8.15)$$

and

$$\tilde{W}' = (1 - g) \tilde{R}' + \tilde{D}' \quad (8.16)$$

From Equations (8.11), (8.12), (8.15) and (8.16), we know

$$\begin{aligned} E_0(\tilde{Y}'_t) &= \bar{R}' + \bar{D}' \\ &= \bar{Y}' \\ &= \text{constant} \quad \text{for } 1 \leq t \leq \infty \end{aligned} \quad (8.17)$$

$$\begin{aligned} E_0(\tilde{W}'_t) &= (1 - g) \bar{R}' + \bar{D}' \\ &= \bar{W}' \\ &= \text{constant} \quad \text{for } 1 \leq t \leq \infty \end{aligned} \quad (8.18)$$

Let

$$\theta' = B' / (B' + S')$$

denote the ratio of bond value to firm value if the project is not adopted.

Define

$$K' = \bar{W}' / (B' + S') \quad (8.19)$$

Then,

$$\begin{aligned} K'(B' + S') &= \bar{W}' \\ &= (1 - g) \bar{R}' + \bar{D}' \quad (\text{from Equation (8.18)}) \\ &= (1 - g) K'_B B' + K'_S S' \quad (\text{from (8.13) and (8.14)}) \end{aligned}$$

Hence,

$$\begin{aligned} K' &= (1 - g) K'_B B' / (B' + S') + K'_S S' / (B' + S') \\ &= (1 - g) \theta' K'_B + (1 - \theta') K'_S \end{aligned} \quad (8.20)$$

Assume that $\tilde{R}$ and $\tilde{D}$ satisfy the assumptions made in the Cost-of-Capital model, i.e., Equations (8.4) through (8.6). Furthermore, we assume

$$E_0(\tilde{R}_t) = \bar{R} = \text{constant} \quad \text{for } 1 \leq t \leq \infty \quad (8.21)$$

$$E_0(\tilde{D}_t) = \bar{D} = \text{constant} \quad \text{for } 1 \leq t \leq \infty \quad (8.22)$$

Equation (8.21) says that the time-0 expectations of all the future bond payments are identical with each other. Equation (8.22) says that the time-0 expectations of all the future dividend payments are identical with each other. Both equations are referred to the situation in which the project

is adopted. Then,

$$B = \sum_{t=1}^{\infty} \frac{\bar{R}}{(1 + K_B)^t} = \frac{\bar{R}}{K_B} \quad (8.23)$$

$$S = \sum_{t=1}^{\infty} \frac{\bar{D}}{(1 + K_S)^t} = \frac{\bar{D}}{K_S} \quad (8.24)$$

where B = market value of all current and new bonds
if the project is adopted,

S = market value of all current and new shares
if the project is adopted,

K_B = cost of capital of $\tilde{R}$, and

K_S = cost of capital of $\tilde{D}$.

Let $\tilde{X}_t$, $\tilde{DP}_t$, and $\tilde{I}_t$ be the cash earnings before interest and tax expenses, the depreciation expenses, and the net investment (=investment - disinvestment), respectively, in period t , if the project is adopted. Equations (1.1) and (1.2) tell us that

$$\tilde{Y} = \tilde{R} + \tilde{D} = \left[(1 - g)\tilde{X} + g\tilde{DP} - \tilde{I} \right] + g\tilde{R}$$

where g = tax rate. Let

$$\tilde{W} = (1 - g)\tilde{X} + g\tilde{DP} - \tilde{I}$$

We can interpret $\tilde{W}$ as the stream of future cash flows to all current and new shareholders of the unlevered (i.e., all-equity) firm if the project is adopted. Then,

$$\tilde{Y} = \tilde{R} + \tilde{D} = \tilde{W} + g \tilde{R} \quad (8.25)$$

and

$$\tilde{W} = (1 - g) \tilde{R} + \tilde{D} \quad (8.26)$$

From Equations (8.21), (8.22), (8.25) and (8.26), we know

$$\begin{aligned} E_0(\tilde{Y}_t) &= \bar{R} + \bar{D} \\ &= \bar{Y} \\ &= \text{constant} \quad \text{for } 1 \leq t \leq \infty \end{aligned} \quad (8.27)$$

$$\begin{aligned} E_0(\tilde{W}_t) &= (1 - g) \bar{R} + \bar{D} \\ &= \bar{W} \\ &= \text{constant} \quad \text{for } 1 \leq t \leq \infty \end{aligned} \quad (8.28)$$

Let

$$\theta = B / (B + S)$$

denote the ratio of bond value to firm value if the project is adopted.

Define

$$K = \bar{W} / (B + S) \quad (8.29)$$

Then,

$$\begin{aligned} K(B + S) &= \bar{W} \\ &= (1 - g) \bar{R} + \bar{D} \quad (\text{from Equation (8.28)}) \\ &= (1 - g) K_B B + K_S S \quad (\text{from (8.23) and (8.24)}) \end{aligned}$$

Hence,

$$\begin{aligned} K &= (1 - g) K_B B / (B + S) + K_S S / (B + S) \\ &= (1 - g) \theta K_B + (1 - \theta) K_S \quad (8.30) \end{aligned}$$

Let

$$\Delta \tilde{W} = \tilde{W} - \tilde{W}' \quad (8.31)$$

From Equations (8.18) and (8.28), we know

$$\begin{aligned} E_0(\Delta \tilde{W}_t) &= \bar{W} - \bar{W}' \\ &= \Delta \bar{W} \\ &= \text{constant} \quad \text{for } 1 \leq t \leq \infty \quad (8.32) \end{aligned}$$

Note that

$$\begin{aligned} \Delta \tilde{W} &= \tilde{W} - \tilde{W}' \\ &= [(1 - g) \tilde{X} + g \tilde{DP} - \tilde{I}] - \\ &\quad [(1 - g) \tilde{X}' + g \tilde{DP}' - \tilde{I}'] \\ &= (1 - g)(\tilde{X} - \tilde{X}') + g(\tilde{DP} - \tilde{DP}') - (\tilde{I} - \tilde{I}') \\ &= (1 - g)\Delta \tilde{X} + g\Delta \tilde{DP} - \Delta \tilde{I} \quad (8.33) \end{aligned}$$

$$\text{where } \Delta \tilde{X} = \tilde{X} - \tilde{X}'$$

$$\Delta \tilde{DP} = \tilde{DP} - \tilde{DP}'$$

$$\Delta \tilde{I} = \tilde{I} - \tilde{I}'$$

and

$$\Delta \tilde{Y} = \tilde{Y} - \tilde{Y}'$$

$$= \left[(1 - g) \tilde{X} + g \tilde{DP} - \tilde{I} + g \tilde{R} \right] -$$

$$\left[(1 - g) \tilde{X}' + g \tilde{DP}' - \tilde{I}' + g \tilde{R}' \right]$$

$$= (1 - g) \Delta \tilde{X} + g \Delta \tilde{DP} - \Delta \tilde{I} + g \Delta \tilde{R} \quad (8.34)$$

$$\text{where } \Delta \tilde{R} = \tilde{R} - \tilde{R}'$$

Notice that $\Delta \tilde{Y}$ is the prospect of the project, and $\Delta \tilde{W}$ would be the prospect of the project if the firm were unlevered. It is obvious that

$$\Delta \tilde{Y} = \Delta \tilde{W} + g \Delta \tilde{R} \quad (8.35)$$

and

$$\begin{aligned} E_0(\Delta \tilde{Y}_t) &= \Delta \bar{Y} \\ &= \text{constant} \quad \text{for } 1 \leq t \leq \infty \end{aligned} \quad (8.36)$$

Now we assume

$$K_B' = K_B \quad (8.37)$$

$$K_S' = K_S \quad (8.38)$$

$$\theta' = \theta \quad (8.39)$$

Equation (8.37) says that $\tilde{R}'$ and $\tilde{R}$ have the same risk.

Equation (8.38) says that $\tilde{D}'$ and $\tilde{D}$ have the same risk.

Equation (8.39) says that the firm maintains a fixed ratio of bond value to firm value at time 0 no matter if the project is adopted or not. From Equations (8.37) through (8.39), we know

$$K' = K \quad (8.40)$$

Let ΔV denote the change of the firm value due to the adoption of the project, i.e.,

$$\Delta V = (B + S) - (B' + S')$$

Then,

$$\Delta V = \bar{W}/K - \bar{W}'/K' \quad (\text{from Equations (8.19) and (8.29)})$$

$$= (\bar{W} - \bar{W}')/K \quad (\text{from Equation (8.40)})$$

$$= \Delta \bar{W}/K \quad (\text{from Equation (8.32)})$$

Under the VAP, we know $V_{\Delta Y} = \Delta V$. Hence,

$$V_{\Delta Y} = \Delta \bar{W}/K \quad (8.41)$$

where
$$\Delta \bar{W} = E_0 \left[(1 - g) \Delta \tilde{X}_t + g \Delta \tilde{D}\tilde{P}_t - \Delta \tilde{I}_t \right]$$

$$= \text{constant} \quad \text{for } 1 \leq t \leq \infty$$

$$K = (1 - g) \theta K_B + (1 - \theta) K_S$$

The denominator in Equation (8.41), K , is the so-called weighted average cost of capital. Readers should notice that when the weighted average cost of capital is used as a discount factor to derive $V_{\Delta Y}$, it is $E_0 \left[(1 - g) \Delta \tilde{X}_t + g \Delta \tilde{D}\tilde{P}_t - \Delta \tilde{I}_t \right]$, not $E_0 \left[(1 - g) \Delta \tilde{X}_t + g \Delta \tilde{D}\tilde{P}_t - \Delta \tilde{I}_t + g \Delta \tilde{R}_t \right]$, which will be discounted.

To derive Equation (8.41), we have made the following assumptions:

- (1) $\tilde{R}'$ and $\tilde{D}'$ satisfy Equations (8.4) through (8.6),
which are the 3 assumptions made in the Cost-of-Capital model.
- (2) $E_0(\tilde{R}'_t) = \text{constant}$
 $E_0(\tilde{D}'_t) = \text{constant} \quad \text{for } 1 \leq t \leq \infty$
- (3) $\tilde{R}$ and $\tilde{D}$ satisfy Equations (8.4) through (8.6).
- (4) $E_0(\tilde{R}_t) = \text{constant}$
 $E_0(\tilde{D}_t) = \text{constant} \quad \text{for } 1 \leq t \leq \infty$
- (5) $K_B = K'_B$, and $K_S = K'_S$.
- (6) $\theta = \theta'$

These assumptions are very restrictive. We have to question the implication and the prediction of Equation (8.41) when it is applied to most real cases. However, once managers have decided to use Equation (8.41), it can be applied to both projects and mutually exclusively alternatives.

Two relevant comments are made below.

Comment 1 The assumption of θ being equal to θ' says that the firm maintains a fixed ratio of bond value to firm value at time 0 no matter if the project is adopted or not. This assumption does not guarantee that the same ratio will prevail in the future even if no change in the mix of projects is made in the future. To see this, let us suppose that the project is adopted, let τ denote some time in the future, and let

$\tilde{B}_\tau$ = time- τ market value of $\{\tilde{R}_{\tau+1}, \tilde{R}_{\tau+2}, \dots, \tilde{R}_\infty\}$

$\tilde{S}_\tau$ = time- τ market value of $\{\tilde{D}_{\tau+1}, \tilde{D}_{\tau+2}, \dots, \tilde{D}_\infty\}$

The assumption that $\tilde{R} = [\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_\infty]$ and $\tilde{D} = [\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_\infty]$ satisfy Equations (8.4) through (8.6) tells us that

$$\tilde{B}_\tau = \sum_{t=\tau+1}^{\infty} \frac{\tilde{E}_\tau(\tilde{R}_t)}{(1 + K_B)^{t-\tau}} \quad (8.42)$$

$$\tilde{S}_\tau = \sum_{t=\tau+1}^{\infty} \frac{\tilde{E}_\tau(\tilde{D}_t)}{(1 + K_S)^{t-\tau}} \quad (8.43)$$

Since we are now at time 0, we regard $\tilde{B}_\tau$, $\tilde{S}_\tau$, $\tilde{E}_\tau(\tilde{R}_t)$ and $\tilde{E}_\tau(\tilde{D}_t)$ as random variables. Recall that at time 0

$$\begin{aligned} B &= \sum_{t=1}^{\infty} \frac{E_0(\tilde{R}_t)}{(1 + K_B)^t} \\ &= \sum_{t=1}^{\infty} \frac{\bar{R}}{(1 + K_B)^t} \\ &= \frac{\bar{R}}{K_B} \end{aligned} \quad (8.44)$$

$$S = \sum_{t=1}^{\infty} \frac{E_0(\tilde{D}_t)}{(1 + K_S)^t}$$

$$\begin{aligned}
&= \sum_{t=1}^{\infty} \frac{\bar{D}}{(1 + K_S)^t} \\
&= \frac{\bar{D}}{K_S}
\end{aligned} \tag{8.45}$$

From Equations (8.42) through (8.45), it is obvious that $\tilde{B}_\tau / (\tilde{B}_\tau + \tilde{S}_\tau)$ is not necessarily equal to $B / (B + S) = \theta$.

A set of sufficient conditions for $\tilde{B}_\tau / (\tilde{B}_\tau + \tilde{S}_\tau)$ to be equal to $B / (B + S)$ contains

$$\tilde{E}_\tau(\tilde{R}_{\tau+1}) = \tilde{E}_\tau(\tilde{R}_{\tau+2}) = \dots = \tilde{E}_\tau(\tilde{R}_\infty) = \widetilde{R}(\tau) \tag{8.46}$$

$$\tilde{E}_\tau(\tilde{D}_{\tau+1}) = \tilde{E}_\tau(\tilde{D}_{\tau+2}) = \dots = \tilde{E}_\tau(\tilde{D}_\infty) = \widetilde{D}(\tau) \tag{8.47}$$

and

$$\frac{\widetilde{R}(\tau)}{\widetilde{D}(\tau)} = \frac{\bar{R}}{\bar{D}} \tag{8.48}$$

One interpretation of Equation (8.46) is given as follows: At time τ , investors will revise their expectations, on the basis of the information available at time τ , about all the subsequent bond payments. Equation (8.46) requires that the time- τ expectations of all the subsequent bond payments be identical with each other. Notice that the time- τ expectations are random variables with reference to time 0. A similar interpretation can also be drawn for Equation (8.47).

One interpretation of Equation (8.48) is given as follows:
 No matter how investors revise their expectations at time τ about subsequent bond payments and dividend payments (but still in the way specified by Equations (8.46) and (8.48)), the ratio of time- τ expectation of subsequent bond payments to time- τ expectation of subsequent dividend payments is required by Equation (8.48) to remain fixed at $\bar{R}/\bar{D}$, the ratio of time-0 expectation of future bond payments to time-0 expectation of future dividend payments. Then,

$$\tilde{B}_\tau = \widetilde{\bar{R}(\tau)} / K_B \quad (8.49)$$

$$\tilde{S}_\tau = \widetilde{\bar{D}(\tau)} / K_S \quad (8.50)$$

and

$$\frac{\tilde{B}_\tau}{\tilde{B}_\tau + \tilde{S}_\tau} = \frac{B}{B + S} = \theta \quad (8.51)$$

Now if we assume that Equations (8.46) through (8.48) hold for any time in the future, then the ratio of bond value to firm value will remain fixed at $B / (B + S)$ in the future. Notice that both $\widetilde{\bar{R}(\tau)}$ and $\widetilde{\bar{D}(\tau)}$ can change through time τ . But their ratio is required to remain fixed by Equation (8.48).

Now suppose that we directly assume

$$\frac{\tilde{B}_\tau}{\tilde{B}_\tau + \tilde{S}_\tau} = \frac{B}{B + S} = \theta \quad (8.52)$$

for any time τ in the future. Then, Equation (8.52), together with Equations (8.46) and (8.47), will lead us to Equation (8.48). That is to say, if the ratio of bond value to firm value is kept fixed in the future, then the ratio of revised expectation of subsequent bond payments to revised expectation of subsequent dividend payments has to remain fixed as long as Equations (8.46) and (8.47) hold.

Comment 2 To derive Equation (8.41), we assume that the Cost-of-Capital model is appropriate for $\tilde{R}$ and $\tilde{D}$. This does not necessarily imply that the Cost-of-Capital model is also appropriate for $\tilde{Y} = \tilde{R} + \tilde{D}$. Specifically, $\tilde{Y}$ does not necessarily satisfy Equation (8.4) even if $\tilde{R}$ and $\tilde{D}$ do. To see this, let $\tilde{e}_{t\tau}^Y$, $\tilde{e}_{t\tau}^R$ and $\tilde{e}_{t\tau}^D$ be such that

$$\tilde{E}_\tau(\tilde{Y}_t) = E_{\tau-1}(\tilde{Y}_t)(1 + \tilde{e}_{t\tau}^Y)$$

$$E_{\tau-1}(\tilde{e}_{t\tau}^Y) = 0$$

$$\tilde{E}_\tau(\tilde{R}_t) = E_{\tau-1}(\tilde{R}_t)(1 + \tilde{e}_{t\tau}^R)$$

$$E_{\tau-1}(\tilde{e}_{t\tau}^R) = 0$$

$$\tilde{E}_\tau(\tilde{D}_t) = E_{\tau-1}(\tilde{D}_t)(1 + \tilde{e}_{t\tau}^D)$$

$$E_{\tau-1}(\tilde{e}_{t\tau}^D) = 0$$

for all $\tau \leq t$ and $1 \leq t \leq \infty$. Since $\tilde{R}$ and $\tilde{D}$ satisfy Equation (8.4), we have

$$\text{cov}(\tilde{e}_{t\tau}^R, \tilde{r}_{M\tau}) = \text{constant}$$

$$\text{cov}(\tilde{e}_{t\tau}^D, \tilde{r}_{M\tau}) = \text{constant}$$

for all $\tau \leq t$ and $1 \leq t \leq \infty$. Then,

$$\begin{aligned} \text{cov}(\tilde{e}_{t\tau}^Y, \tilde{r}_{M\tau}) &= \frac{\text{cov}(\tilde{E}_\tau(\tilde{Y}_t), \tilde{r}_{M\tau})}{E_{\tau-1}(\tilde{Y}_t)} \\ &= \frac{\text{cov}(\tilde{E}_\tau(\tilde{R}_t + \tilde{D}_t), \tilde{r}_{M\tau})}{E_{\tau-1}(\tilde{Y}_t)} \\ &= \frac{\text{cov}(\tilde{E}_\tau(\tilde{R}_t), \tilde{r}_{M\tau})}{E_{\tau-1}(\tilde{Y}_t)} + \frac{\text{cov}(\tilde{E}_\tau(\tilde{D}_t), \tilde{r}_{M\tau})}{E_{\tau-1}(\tilde{Y}_t)} \\ &= \frac{\text{cov}(\tilde{e}_{t\tau}^R, \tilde{r}_{M\tau}) E_{\tau-1}(\tilde{R}_t)}{E_{\tau-1}(\tilde{Y}_t)} + \frac{\text{cov}(\tilde{e}_{t\tau}^D, \tilde{r}_{M\tau}) E_{\tau-1}(\tilde{D}_t)}{E_{\tau-1}(\tilde{Y}_t)} \end{aligned}$$

Obviously, $\text{cov}(\tilde{e}_{t\tau}^Y, \tilde{r}_{M\tau})$ is not necessarily a constant for all $\tau \leq t$ and $1 \leq t \leq \infty$. To make $\text{cov}(\tilde{e}_{t\tau}^Y, \tilde{r}_{M\tau})$ a constant for all $\tau \leq t$ and $1 \leq t \leq \infty$, we need

$$\frac{E_{\tau}(\tilde{R}_t)}{E_{\tau}(\tilde{D}_t)} = \text{constant} \quad (8.59)$$

for all $\tau \leq t$ and $1 \leq t \leq \infty$. Hence, if the Cost-of-Capital model is appropriate for $\tilde{R}$ and $\tilde{D}$ and Equation (8.59) holds, then the Cost-of-Capital model is also appropriate for $\tilde{Y} = \tilde{R} + \tilde{D}$.

It is interesting to know that Equations (8.46) through (8.48) imply Equation (8.59); or Equations (8.46), (8.47) and (8.52) also imply Equation (8.59).

Similarly, the assumption that $\tilde{R}$ and $\tilde{D}$ satisfy Equation (8.4) does not necessarily imply that $\tilde{W} = (1 - g) \tilde{R} + \tilde{D}$ satisfies Equation (8.4). Again, Equation (8.59) can make $\tilde{W}$ satisfying Equation (8.4) and hence make the Cost-of-Capital model appropriate for $\tilde{W}$.

In the above, Comment 1 and Comment 2 are explained in terms of the situation where the project is adopted. These two comments are also applicable to the situation where the project is not adopted.

SECTION 9

TIME-STATE-PREFERENCE MODEL IN MULTIPERIOD PROJECT EVALUATION

The marginal utility one dollar can bring to an investor at time t if he is a millionaire at time t is quite different from the marginal utility one dollar can bring to him at time t if he is almost starved to death at time t . The current value of one dollar at time t depends not only on the time-length from now to time t , but also on the state occurring at time t . This comment brings us to the Time-State-Preference (TSP) model.

The following definitions will be used:

(s, t) = the state s at time t

e_{st} = the current (time 0) price of a claim to a
payoff of \$1 at time t if state s occurs at time t

p_{st} = the probability that state s occurs at time t

$n(t)$ = the number of possible states at time t

The TSP model assumes that people agree on the states obtainable and agree on the e_{st} magnitudes. However, people may agree or disagree on the p_{st} magnitudes. People are said

to hold homogeneous expectations if they agree on the p_{st} magnitudes, and to hold heterogeneous expectations otherwise.

Let Z_{st} denote the cash flow at time t if state s occurs at time t . Then, a prospect (i.e., a sequence of future cash flows) $\tilde{Z}$ can be fully described by $[Z_{11}, Z_{21}, \dots, Z_{n(1)1}; Z_{12}, Z_{22}, \dots, Z_{n(2)2}; \dots; Z_{1T}, Z_{2T}, \dots, Z_{n(T)T}]$, where T is the last period of the relevant horizon.

The TSP model assumes that each individual assigns a (subjective) probability p_{st} to each (s, t) such that

$$p_{1t} + p_{2t} + \dots + p_{n(t)t} = 1 \quad \text{for all } t \quad (9.1)$$

In perfect capital markets where the VAP holds because of arbitrage, we have

$$\begin{aligned} V_Z &= \sum_t \sum_s V_{Z_{st}} \\ &= \sum_t \sum_s e_{st} Z_{st} \quad \text{in equilibrium} \end{aligned} \quad (9.2)$$

Equation (9.2) can hold in some other types of capital markets. Nevertheless, we shall assume Equation (9.2) holds in equilibrium throughout our discussion. Then, if we know e_{st} , we can compute V_Z .

The prospect $\tilde{Z}$ is a $\sum_{t=1}^T n(t)$ dimensional vector. If

we can find $\sum_{t=1}^T n(t)$ linearly independent prospects, which are publicly traded in perfect efficient capital markets, together with their current prices, then we obtain $\sum_{t=1}^T n(t)$ equations of the form (9.2) and e_{st} ($t = 1, 2, \dots, T$; $s = 1, 2, \dots, n(t)$) can be uniquely determined by solving a system of $\sum_{t=1}^T n(t)$ simultaneous equations. Under the assumptions that people agree on the states obtainable and agree on the e_{st} magnitudes, and that Equation (9.2) holds in equilibrium, the e_{st} magnitudes resulting from solving the preceding system of $\sum_{t=1}^T n(t)$ simultaneous equations will not vary when a different set of $\sum_{t=1}^N n(t)$ linearly independent prospects is used. After we have obtained e_{st} , we can use Equation (9.2) to obtain V_z . An example is given below. Table 9-1 gives all relevant information.

(s, t)	<u>Cash Flows</u>			<u>Price</u>
	<u>(1, 1)</u>	<u>(1, 2)</u>	<u>(2, 2)</u>	
Security A	\$40	\$70	\$10	\$49
Security B	\$10	\$ 2	\$ 5	\$ 5
Security C	\$10	\$10	\$10	\$10
$\tilde{z}$	\$60	\$74	\$20	?

TABLE 9-1

Solve the following system of 3 simultaneous equations

$$40 e_{11} + 70 e_{12} + 10 e_{22} = 49$$

$$10 e_{11} + 2 e_{12} + 5 e_{22} = 5$$

$$10 e_{11} + 10 e_{12} + 10 e_{22} = 10$$

We have

$$e_{11} = 0.3$$

$$e_{12} = 0.5$$

$$e_{22} = 0.2$$

Using Equation (9.2), we obtain

$$V_Z = (60)(0.3) + (74)(0.5) + (20)(0.2) = 59$$

An easier way to obtain V_Z exists if $\tilde{Z}$ is spanned by other prospects which are publicly traded in a perfect efficient capital market. Suppose that $\tilde{Z}_i$ ($i = 1, 2, \dots, m$) are publicly traded in perfect efficient capital markets. Suppose $\tilde{Z}$ is spanned by $\tilde{Z}_i$ ($i = 1, 2, \dots, m$); that is to say, there exist m constants a_i ($i = 1, 2, \dots, m$) such that

$$\tilde{Z} = \sum_{i=1}^m a_i \tilde{Z}_i \quad (9.3)$$

Then, the TSP model says that

$$V_Z = \sum_{i=1}^m a_i V_{Z_i} \quad (9.4)$$

It is just another way to state the VAP. Back to our previous example but let $\tilde{Z} = [45; 75, 15]$. Now,

$$[45; 75, 15] = [40; 70, 10] + 0.5[10; 10, 10]$$

$$V_Z = 49 + (0.5)(10) = 54$$

If we use Equation (9.2), then

$$V_Z = (45)(0.3) + (75)(0.5) + (15)(0.2) = 54$$

The concept of the TSP model is simple, while the application of the TSP model is not that simple. The major problem is that in each state we have to give an estimate of cash flow for each prospect used in the system of simultaneous equations, and we may need to construct many states for the TSP model to be accurate in describing and predicting the reality. Recent papers on the TSP model can be found in [5, 28, 29].

The fundamental procedure outlined in Section 1 is still valid when the TSP model is used to evaluate $\Delta\tilde{Y}$. The TSP model can be applied to both projects and mutually exclusive alternatives.

SECTION 10

A NEW LOOK AT

SOME TRADITIONAL APPROACHES OF PROJECT EVALUATION

Several approaches of project evaluation, which have been used by practicing managers or advocated by academics, become questionable in the face of modern developments in finance theory. Three of these questionable approaches will be discussed in this section. These three approaches are

Approach 1: Base case and sensitivity analysis;

Approach 2: Monte Carlo simulation and risk profile; and

Approach 3: The firm's utility function.

In order to discuss these approaches, we have to explain some terms in details.

In evaluating a project, a manager (or a group of managers) is faced with two sets of variables: decision variables and state variables. A manager has control on decision variables while he has no control on state variables. He has to accept whatever state variables turn out to be. We shall assume that each state variable is a random variable. A manager can define an alternative in which

he specifies a particular value for each decision variable. Two alternatives must mutually exclusive, in the sense that the adoption of one precludes the adoption of the other. Associated with each alternative, a set of relevant state variables can be identified. Since we assume that state variables are random variables, there is a joint probability distribution for these relevant state variables. A manager can define a scenario in which he specifies a particular value for each state variable identified. Two scenarios of an alternative must be mutually exclusive, in the sense that the occurrence of one precludes the occurrence of the other.

Approach 1: Base case and sensitivity analysis

In this approach, managers first construct a reference scenario, called the base case scenario, and then construct several other scenarios, each being such that only a state variable (or a group of highly correlated state variables) has its value changed in comparison with the base case scenario. Let $[-I_0, \Delta\tilde{Y}_1, \Delta\tilde{Y}_2, \dots]$ be the cash flows of a project or an alternative within a project. Once a scenario has been defined, $\Delta\tilde{Y}_t$ assumes a particular value, say $\Delta\hat{Y}_t$. For each scenario, define DV as follows:

$$DV = -I_0 + \sum_{t=1}^{\infty} \frac{\Delta\hat{Y}_t}{\prod_{\tau=1}^t (1 + r_{f\tau})} \quad (10.1)$$

where $r_{f\tau}$ = riskless rate of return in period τ .

Different scenarios have different $[\Delta \hat{Y}_1, \Delta \hat{Y}_2, \dots]$ and hence yield different DV. An interpretation of DV is that if the scenario associated with $[\Delta \hat{Y}_1, \Delta \hat{Y}_2, \dots]$ is bound to occur in the real world, then we have a case of certainty and DV becomes the current market value of the project.

The sensitivity analysis is a process of comparing the DV of the base case, in which all the state variables are usually set at their expected values or most likely values, and the DV's of other constructed scenarios. Through the sensitivity analysis, some state variables may be identified as influential in the sense that DV changes a great deal as these influential state variables change their values across scenarios. Managers may decide to gather more information regarding these influential state variables before they make any decision.

This approach has the following shortcomings. First, the modifier "influential" is not well defined. A state variable regarded as influential by one manager may not be regarded as influential by another manager. It may cause a communication problem and an inconsistency problem. Second, the set of state variables to be identified as influential is highly dependent upon the base case scenario chosen at the beginning and other scenarios constructed later during the sensitivity analysis. A different set of scenarios constructed

may result in a different set of influential state variables. Then, to which set should managers pay attention? Third, state variables identified as influential may be actually not influential at all. This can happen because a highly improbable scenario may be somehow constructed which changes DV a great deal, and consequently some state variables are identified as influential. However, since the scenario is so improbable that it is not worthwhile to gather more information on these variables. Fourth, a limited number of scenarios with their respective DV's can not tell the whole story about the current market value of the project. This shortcoming is most serious since the current market value is precisely what the project evaluation looks for.

Sometimes the cost of capital, instead of the riskless rate of return, is used in Equation (10.1) to obtain DV. For the purpose of identifying influential states variables, the use of the cost of capital is neither better nor worse than the use of the riskless rate of return. Conceivably, the use of a different discount factor in Equation (10.1) may result in a different set of influential state variables. However, because of the inherent shortcomings of this approach, it can not be told which discount factor will result in the more correct set of influential state variables. However, readers are reminded that when we talk about the cost of capital of a project, we implicitly assume that the project

satisfies Equations (8.4) through (8.6). For a project which fails to satisfy Equations (8.4) through (8.6), there exists no single factor, called the cost of capital, which can be used to discount the expected cash flows. Consequently, only if Equations (8.4) through (8.6) are satisfied, we can possibly say that we are using the cost of capital in Equation (10.1).

Approach 2: Monte Carlo simulation and risk profile

Since Approach 1 only constructs a limited number of scenarios, it is wondered if an approach constructing all possible scenarios will perform better than Approach 1. When the interrelationships among state variables are complex, it is difficult to construct all possible scenarios. When at least one state variable is continuous, it becomes impossible to construct all scenarios. Hence, a technique called the Monte Carlo simulation is used. Essentially, this technique constructs various scenarios with the aid of computers and the user can specify the number of scenarios to be constructed. Again, the DV in Equation (10.1) is calculated for each scenario constructed during the simulation. The output of the simulation is a risk profile of DV, which is a relative frequency tableau (or plot) for DV. When the number of scenarios constructed in the simulation is large, the risk profile of DV approximates the probability distribution of $\widetilde{DV}$, where $\widetilde{DV}$ is defined as

$$\widetilde{DV} = -I_0 + \sum_{t=1}^{\infty} \frac{\widetilde{Y}_t}{\prod_{\tau=1}^t (1 + r_f \tau)} \quad (10.2)$$

Sensitivity analysis can also be conducted by holding one or more state variables at some fixed values and simulating other state variables. In this way, managers may be able to identify some state variables as influential.

This approach has the following shortcomings. First, the first three shortcomings mentioned in Approach 1 are also applicable here. Second, even if the resulting risk profile of DV is a good approximation of the true probability distribution of $\widetilde{DV}$, it still can not tell the current market value of $[-I_0, \Delta \widetilde{Y}_1, \Delta \widetilde{Y}_2, \dots]$. Modern finance theory establishes the current market value of a project through the co-movement between the cash flows of the project and the market rates of return; it offers no link between the risk profile of DV and the current market value.

Notice that Approach 1 and Approach 2 are not totally futile in the face of the modern finance theory. On the contrary, these two approaches can be very useful if they are applied in the right direction. Take the one-period CAFM for example. To decide $V_{\Delta Y}$, we have to find $\text{cov}(\Delta \widetilde{Y}, \widetilde{r}_M)$ first.

$\Delta\tilde{Y}$ and $\tilde{r}_M$ are correlated with each other by way of many underlying economywide variables such as the rate of inflation, the growth rate of real GNP, changes in tax law and so on. Each economywide variable can have a different impact on $\Delta\tilde{Y}$ and $\tilde{r}_M$. Their interrelationships can be complicated. It may become highly difficult, if not impossible, for an analytical approach to obtain $\text{cov}(\Delta\tilde{Y}, \tilde{r}_M)$. The idea of base case and sensitivity analysis can be used here to identify those influential (however defined) economywide variables and managers may decide to gather more information about the relationship between $\Delta\tilde{Y}$ (or $\tilde{r}_M$) and the influential economywide variables. Also, Monte Carlo simulation can yield a sequence of pairs of ΔY and r_M , from which an estimate of $\text{cov}(\Delta\tilde{Y}, \tilde{r}_M)$ is obtainable. Readers should be reminded that if some economywide variables are not independent of each other, their dependencies should be incorporated into the simulation program. Treating dependent variables as independent will distort the reality of $\text{cov}(\Delta\tilde{Y}, \tilde{r}_M)$. The preceding example tells us that both Approach 1 and Approach 2 can still survive the modern developments in finance theory if they are used in the right direction.

Approach 3: The firm's utility function

A firm's utility function, as used in this approach, is essentially a policy statement of a firm, which formalizes the

managers' judgement so that they can exercise their judgement consistently. Faced with the probability distribution of DV (or internal rate of return), managers do not know exactly what to do with it. Faced with the same distribution, two managers may make quite different decisions or the same manager may make quite different decisions when he is in different moods. The firm's utility function is then set up to rectify this inconsistency. Usually, with the help of outside consultants, members of the top decision-making unit of the firm reach an agreed-upon utility function for DV (or internal rate of return) either in open conflict or by compromise. This utility function is then used as a tool of project evaluation in the firm. The alternative with a higher certainty equivalent than any other alternative within the same project will be chosen to represent the project. If the certainty equivalent of a project exceeds some minimum level (which is also a firm's policy), the project is adopted.

The only merit of this approach is to help managers reach a consistent decision. However, a consistent decision is not necessarily a good decision. In the worst case, this approach can consistently yield bad decisions. As we know, managers should make decisions to maximize the market value of current shares. If the members of the top decision-making unit do not keep this Market Value Maximization rule in their mind, the resulting utility function obviously will not help managers make decisions to maximize the market value of current

shares. Even if the members of the top decision-making unit do keep the Market Value Maximization rule in their mind, it is still not sure whether the resulting utility function is consistent with the Market Value Maximization rule. As mentioned in Approach 1 and Approach 2, there is no link between DV (or internal rate of return) and the market value of current shares. Consequently, DV (or internal rate of return) can not have a concrete meaning to the members of the top decision-making unit, in terms of market value of current shares. Then, how can they be expected to give meaningful utility measures to DV (or internal rate of return)?

The firm's utility function can be even fancier by including other profitability measures, e.g., market shares, as its arguments. However, its shortcomings remain unchanged.

So far we have discussed three questionable approaches which have been used or advocated by various people. All these discussions are based upon the assumption, among others, that there exist perfect and efficient (or nearly perfect and efficient) capital markets and hence a better approach, namely the CAPM, exists. Readers should keep in mind that these discussions may not be valid in many countries where the capital markets are by no means perfect and efficient. For these countries, the three approaches discussed above may be their only choices. But in the United States of America where the capital markets are much closer to being perfect and efficient, managers are advised to try to use the CAPM in evaluating projects. Each of the three approaches discussed above can be used as a supplementary tool to the CAPM.

SECTION 11

CONCLUSION

This paper has presented a comprehensive reasoning as to how managers of private profit-seeking firms, whose shares are publicly traded in capital markets, should evaluate their candidate projects. Part 1 of this paper obtains a decision rule that given a project with an initial cash outlay I_0 (known with certainty) and a prospect $\Delta\tilde{Y}$ (a vector of random future cash flows), the managers should accept this project if the current market value of the prospect, namely $V_{\Delta Y}$, exceeds its initial cash outlay I_0 . This decision rule is based upon several assumptions, one being that capital markets are perfect, efficient, and in equilibrium. This decision rule leads us to an immediate question: how to determine $V_{\Delta Y}$. The CAPM is then introduced to answer this question. The CAPM delivers a simple idea:

In a one-period horizon, the current market value of any end-of-period cash flow is determined by the co-movement between the cash flow and the aggregate rate of return of all the end-of-period cash flows in the economy.

Equation (4.6) best conveys this idea. The CAPM by no means solves all the problems encountered in getting $V_{\Delta Y}$. First of all, by definition all the parameters used in Equation (4.6) are not historical data; these parameters are associated with the upcoming period, and hence are actually unknown to us. In order to use the CAPM, we have to form a good judgement on these parameters. In the first step, historical data are helpful in getting rough estimates. Unless managers are pretty sure that the history is a good reflection of the future, these estimates should be adjusted for any change under way or anticipated to happen in the relevant future. Regression analysis and scenario approach, as used in this paper, can be helpful in getting these estimates. However, a more important problem arises when we realize that the CAPM is a one-period model, in which the only cash flow is assumed to occur at the end of the period, but a project can have multiple cash flows, which can occur in different points in time. To preserve the significance of timing difference among cash flows on one hand and to obtain a simple model on the other hand, the whole horizon is divided into periods. All the cash flows occurring in a period are assumed to happen at the end of that period. Consequently, we are faced with a multiperiod problem instead of a one-period problem.

Part 2 of this paper primarily concerns itself with the extension of the CAPM to a multiperiod horizon. An

immediate restriction is that if the CAPM is applied to each period of a multiperiod horizon, the CAPM-relevant characteristics of the portfolio opportunity set available in each future period must be known with certainty (nonstochastic) at time 0. This restriction may not be realistic in real life; however, before the CAPM is further refined, this restriction just cannot be ignored if one wishes to apply the CAPM to a multiperiod project. This restriction, together with other numerous assumptions underlying the one-period CAPM, do not necessarily prevent the CAPM from yielding implications reasonably consistent with the observed phenomenon or performing reasonably well in prediction in either a one-period horizon or a multiperiod horizon.

The Certainty Equivalent (CE) model and the Risk-adjusted-Discount (RAD) model, all based upon the CAPM, are equivalent to each other. These two models should yield the same set of implications or predictions when the same set of estimates are used. As Equations (6.1) and (6.2) or Equations (7.1) and (7.2) demonstrate, the difficulty with these two models is that a large number of parameters have to be estimated. To simplify, additional assumptions can be adopted to reduce the number of parameters to be estimated. However, by doing so, only certain types of projects are eligible for using the consequent simplified models. The Cost-of-Capital model, as represented by Equation (8.2) , is one of those simplified models. In spite of

its popularity among practicing managers, the Cost-of-Capital model has some implications which may not be compatible with many projects. This incompatibility has not been widely recognized yet. The use of the weighted average cost of capital, as represented by Equation (8.41), involves so many assumptions that one has to question its implications and predictions when it is applied to many real cases.

The CAPM is by all means one of the most important developments in finance theory. However, this model in its current stage does have its limitation in applications. Researchers shall either improve the CAPM or try to develop another model. The Time-State-Preference (TSP) model is a very promising alternative to the CAPM. However, the TSP model in its current stage also suffers from the fact that a large number of estimates have to be involved in order to get a reasonably accurate answer for $V_{\Delta Y}$. A lot of work is needed before the TSP model reaches its application stage.

Economic evaluation of industrial projects has always been a major challenge facing managers. As more sophisticated theories develop, traditional techniques become questionable. These traditional techniques now need extra assumptions in justifying their use and consequently have their application area diminished. Sophisticated finance theories help us grasping the essence of economic evaluation of industrial projects, but also leave many unsolved questions.

The evolution continues and may be endless. Standing where we are now, the future evolution itself becomes a project. We may ask the following question: is it worthwhile to exchange I_0 for $\Delta\tilde{Y}$, where I_0 is the current effort we put into the studies of project evaluation, and $\Delta\tilde{Y}$ is the prospect of the future benefits we shall receive?

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